Decisiveness of Decision Maker: A Multiple-Self Method

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Abstract

In this article, focusing on goal seeking behavior (i.e., a sequential search process), a two-stage simulation method of elicitation-approximation procedure with the measure of decisiveness has demonstrated, in order to interpret nonlinear probability weighting functions, with cumulative representation of utility, as a decentralized uncertain knowledge model of decision makers. The notion of decisiveness used in the first stage of our method to elicit nonlinear probability weight to fit the experimental patterns of dynamic allocation. And in the later stage, the elicited weight to be approximated with consonant beliefs, so as to interpreted as the distributed beliefs. The nonlinear cumulative probability weighting functions, such as monotone convex / concave capacity, or inverse S shaped function, its curvature changes from concave to convex, have been extensively examined in recent literature. Although cumulative representation of separable utility models, such as rank-dependent expected utility (RDEU) or cumulative prospect theory (CPT), are regarded as the quantitative representation of decision maker with uncertain knowledge base (or evidential corpus), it is vulnerable to describe hedging behavior under linear utility weight. I suggested a way to expand these separable models by clarification of its relation to distributed knowledge "on the spot" of decision points. For more concrete grasp of theoretical concepts, some experimental data has been collected from the iterative multi-choice test, a web-based experimentation system I developed, where students permit to bet his/ her own initial endowments to judge their choice partially (pJudge) unless the endowments vanish, and used in order to apply these decision models to evaluate imperfect knowledge of my students. Technically, I demonstrated several ways of computer simulation, assuming consonant beliefs about stopping time, Mobius inversion of Dempster-Shafers's belief function, spreadsheet models and GA optimization, tree induction, and so on.

1. Introduction

In real life, usually a decision maker is complex cognitive-emotional system with conflicting goals and imperfect knowledge about the future courses of action by itself and by others. Sometimes there is neither perfect satisfaction nor regret-free result of attainable (i.e., the best), and therefore we must be dealing with going?concern. In other words, because misspecifications of problem and unforeseen contingencies are inevitable in our cognitive-social life and also we aware of it, some reliable prescriptive way to repair our irrationality, besides rational choice theories, such as expected utility theory and game theory, cognitive psychology, management science, or

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information systems support decision makers, are needed.

In this regard, it seems me that information acquisition by mixture of knowledge and search with subtle objectives satisfying us by means of making mental representations (i.e., "framing" in terms of Tversky and Kahneman), as well as economic activities with concrete objectives by means of monetary transaction, is noteworthy and is stimulus of inquiry because of its potential to improve irrationality not only with normative lecturing ('Should be rational!'), but with prescriptive way to design remedy against it.

Multiple-self, that regards a single decision maker, who usually tend to violate expected utility theory, as a collection of agents, and optionally with or without their coordination mechanism in style of game theory, is probably one of the most appealing descriptive ways, at least intuitively, in order to model these properties pertain to bounded rational decision makers. Precommitment, the notion repeatedly mentioned in literature, the personal rules of various constraints against decision maker's weak will power, in order to remedy conflicting mind. The analogy of bargaining game, in Schelling's own essay, the analogy of reputation game in Ainslie's model of addiction, as well as the dynamic inconsistency of nonexpected utility maximizer in Stortz's paper. (If you want to know concrete contents of these models, confer Elster (1986) and Ainslie (1992).) Mental account (Thaler, 1990) is another type of precommitment mechanism, the personal rules to manage money in distinctive budgets.

My proposal is a two-stage method to elicit cumulative probability weight of decision makers in sequential choice, and to translate it into (possibly decentralized) uncertain knowledge systems by linear combination of approximated consonant belief function and its conjugate plausibility function. In first-stage of this method, the notion of decisiveness, which can be regarded as analogy of precommitment in multiple-self models, plays the role of keystone to elicit probability weight in accordance with counterfactual reasoning of decision makers.

In this paper, focusing on the situation of experimental multi-choice test I developed, a practical application of our theory to computer-aided web learning on Internet, where students permit to bet his/ her own initial endowments to judge their choice partially ("pJudge" for short) unless the endowments vanish, I tried to apply and verify these decision models to evaluate imperfect knowledge base of my students via computational method. Experimental data has collected in 24 Apr and during 25 Apr to 2 May 2001 of students of my two classes. In our approach, it is assumed that decision makers (i.e., students of my class) have "consonant" beliefs about when the right answer will be found, i.e., the stopping time of search process to find the right answer of each question with

5 branches respectively.

Consonant beliefs are beliefs of events such that there are only nested focal elements have positive basic probability mass. Approximated beliefs may remain some small positive mass out of nested monotone (convex) beliefs. With a little surprise, this can be used to approximate inverse S shaped cumulative weight with linear combination of its conjugate function by means of Mobius inversion techniques, which is also familiar to the researchers in field of belief function and its application to decision theory.

The reminder of this paper follows: In the next 2 sections, related works with the notion of decisiveness including multiple-self, Choquet representation, belief function, and probability weighting function are briefly reviewed. We present the iterative multi-choice test to be analyzed by cumulative representation and probability weighting function approximately in section 4. (In Japanese) RDEU / CPT models and inverse S shaped probability weight are translated, but informally, into the decentralized search process of decision maker with uncertain knowledge base (or evidential corpus). In section 5, I will report with the some experimental data in my class, and simulation results using spreadsheet models and optimization tools (e.g., standard SOLVER addin of Microsoft 's EXCEL and Palisade 's EVOLVER a genetic algorithm (GA) addin to EXCEL) and standard data mining tools such as tree (or rule) induction algorithm that are both familiar to academic users and business decision makers, to induce probability weight and elicit partial knowledge base.

2. Related works and motivational background

The idea of game theoretic formulation for multiple-self decision maker is not new in literature on non-expected utility theory (Strotz, 1956; Karni and Safra, 1989, 1990). And recently absentmindedness of game player with imperfect recall has draw attention of many game theorists. But there is no experimental validation, epistemic foundation, and inductive modeling method of such theories in literature, except for time-preference models in distributed (intertemporal) choice context and Voter 's illusion type experimental study in psychology.

Karni and Safra (1990) proposed the notion of behavioral consistency, distributed agent representation of RDEU nonexpected utility maximizer to apply it to the models of auctions and search. As for search mode, they found the analog of reservation price property under quasi-convex utility functional that is decisive as EU maximizer, and the existence of upper-lower interval of indecisive stopping strategy under quasi-concavity. (See also the last paragraph that refers to Wu 's anxiety model.) In their FIG. 1, p.395, we showed an example of a behaviorally inconsistent decentralized search tree. In spite of their insistence, I think that it is naturally to understand of pass the act "a" to get middle level outcome with the certainty, at the first node, is reasonable enough so that the chance of same or above "with certainty" remains until the sub-node 1 has played. That is somewhat similar advantage to EU maximizer's sequential search (Weitzman, 1979), and preference for flexibility (Nehring, 1999).

Ratifiability (Jeffery, 1983; Harper, 1994), the notion that the decision maker justify his / her choice by best response to their own choice intended, is originating in Richard Jeffery's "Logics of Decision", is another noticeable line of research. Although usually not included in multiple-self context, this game theoretic model of decision maker as game-theoretical mechanism provides a decision theoretical foundation.

Sorting counterfactual beliefs with hidden conditions lead to the commonality of Newcomb problem and several decision paradox, dynamic inconsistency, and preference ladder technique by adding common consequences to induce bounded subaddtitive decision weight.

Besides traditional ego psychology and social psychology, "society of mind" (Minsky, 1986), one of the most stimulative paradigm in recent artificial intelligence research has similar features of multiple-self decision model except for its insisting on the reduction of real intelligent system into relative simpler units and administrative functions of their interaction. Piaget's cognitive equilibrium, or Simon's quasi-decomposable systems are also considered as other lineage of multiple-self.

By the way, in belief-based modeling techniques, both cumulative representations of utility (rank-dependent expected utility (RDEU) or cumulative prospect theory (CPT)) and inverse S shaped probability weight, have been experimentally examined by recent decision science researchers. Especially, capacities (i.e., nonadditive probability weight) of Choquet integral and its relation to the max-min utility representation under multi-probabilities (Gilboa and Schmeidler, 1989; Mukerji, 1997) in these models are utilized to model decision maker's attitude towards uncertainty in probability, or so called ambiguity aversion (under convex capacities and belief functions) or ambiguity seeking (under concave capacities and plausibility functions).

Cumulative representation of separable utility models with nonadditive probabilities or ambiguous beliefs has developed by Quiggin, Schmeidler, Gilboa, Yaari, and many other contributed researchers. In papers on axiomitization of this sort of representation, usually replace independence axiom with comonotonic independence (i.e., ordinal independence) and it does not violate to stochastic dominance. As for decision theory, nonadditive probability models was intended to model ambiguous beliefs and resolve

Ellsberg 's paradox (Ellsberg, 1961) an apparent contradiction to Savage 's Subjective EU, at first, then apply to the game theory, portfolio theory and so on (Dow and Werlang, 1992ab, 1994). But it can also explain besides other type of stylized violations to EU as well as Allais type padadox, those cases which are cannot explained by monotone capacities (i.e., bounded subadditivity).

Inverse S shaped probability weight (Wu and Gonzalez, 1996; Prelec, 1998; Fox and Tversky, 1998; Wu and Gonzalez, 1998; Tversky and Wakker, 1995) shows "from concave to convex" property at probability 0.3 ?- 0.4 (fig. 1), and it can explain two types of marginal effect: certainty effect, tendency to over evaluate probability near to 1 and possibility effect, tendency to under evaluate probability near to 0. Prospect theory by Tversky and Kahneman in risk situations (Kahneman and Tversky, 1979) has extended (including to uncertainty situation) by this type of weight that shows bounded subadditivity Tversky and his collaborators are insisted (i.e., Support Theory), and distinctive weights for gains and losses (Tversky and Kahneman, 1993; Wakker and Tversky, 1993). Recently, new elicitation techniques for biased utility and nonlinear probability weight, with or without the standard sequence, have developed (Abdellaoui, 2000; Bleichrodt and Pianto, 2000; Wakker and Deneffe, 1996).

However, up to middle 1990s, there are several experimental studies reported in 1990 's that researchers did not appreciated descriptive improvement of this type of model against expected utility and other alternative models for risk attitude, and its advantage is limited to only the marginal of Marschak-Machina triangle. It has observed that comonotonic Independence rather tends to be violated (Wu, 1994; Wakker, Erev and Weber, 1994; Mangelsdorff and Weber, 1994; Fennema and Wakker, 1996; Harless and Camerer, 1994; Hey and Orme, 1994).

In spite of its normatively appealing character, so that the cumulative representation of separable utility models with ambiguous beliefs can be regarded as the quantitative representation of decision maker with uncertain knowledge base (or evidential corpus (Smets, 1998)) by using techniques of Dempster and Shafer 's belief function theory (Mukerji, 1997; Mongin, 1994; Jaffrey and Wakker, 1994), it has drawback in descriptive power for various hedging behavior without nonlinear utility weight (see Figure 5.6).

I noticed that rank dependent expected utility model (i.e., anticipated utility of Quiggin) are intended to design imperfection of state and consequence, a la Jeffery, at least partially, and its relation to multiple-self, especially to voter 's illusion type one, or hidden order parameter models with respect to counterfactual reasoning (Leopald and Selten, 1982), which have been repeatedly argued in criticizing papers about decision theoretic foundation of game theory, especially on the implicit assumption of common knowledge and it of rationality of game players (Bicchieri, 1993; Dekel,1997). Relating this line of research to the backward induction puzzle, a famous decision paradox, or Newcomb paradox could have been noticed here, but it may be rather confusing to readers because of the (personal, or conventional?) limit of pages to outlook those research field.

However, the issue has some similarity to nonlinear hedge model in investment science, and I also have a conjecture that it may be solved by computer-based simulation methods. I also will suggest the idea of decisiveness with this approximated consonant beliefs to interpret RDEU and decision weights of decision maker. Kahneman and Varey (1997) has stated the notion of "decisiveness" as exclusive event, in relation to the notion of "propensities" that is the psychological correspondence to probability, which is not accordance with probability in human judgment, such as a counterfactual statement 'He almost won".

The notion of decisiveness by Kahneman and Varey has similar nature to the "minimality" principle, in the sense of Ramsey test and "epistemic entrenchment" in belief revision theory (Gardenfors, 1986) studied by philosophers of language and decision theorists, that the meaning of counterfactual sentence as the "nearest impossible possible world" from the true (or current) possible state of the world (Lewis, 1976). In this paper, the two-stage elicitation-approximation procedure I proposed incorporates it in the first stage, which measures minimal distance of experimental data pattern from the optimal pattern of RDEU maximization as this idea of "nearest impossible possible world". And also in the first stage, this measurement of distance from RDEU-optimaility ranks the series of possible patterns of behavior.

Because of the axiomatization of epistemic entrenchment is identical to consonant belief function, and so as to necessity measure. Even more, it is analogically inferred that the decisive-critical pair of events may be translated into belief function and plausibility function pair, or lower and upper envelope of set of probabilities. But it is meaningless until this analogy can be generalized to inverse S shaped weight by its approximation procedure.

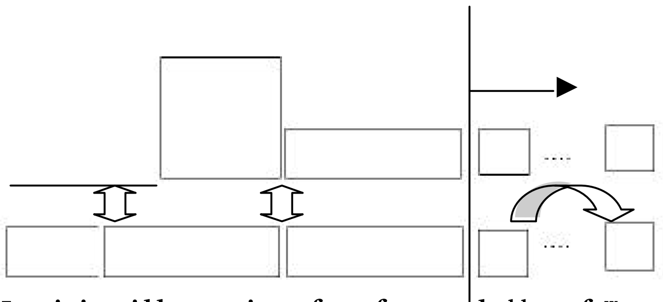


Figure 2.1 Intuitive illustration of preference ladder of Wu and Gonzalez (1996)

Figure 2.1 illustrates how the shift of curvature of weight concave to convex in probability affects attitude of decision maker 's risk preference changing from optimistic to pessimistic. Wu and Gonzalez utilized the fact that, by adding the sequence of same common consequences to a risky gamble and its equivalent but safety gamble, tendency to choose risky is increasing then decreasing. This is called "preference ladder" applied to nonparametric elicitation technique for inverse S shape weight (Wu and Gonzalez, 1996).

Recently George Wu linked iterative RDEU model and nonlinear probability weight to decision maker 's "anxiety" (or thought time based intensity), and it generalizes Bell 's anticipated regret (Wu, 1999). Wu 's anxiety model was intended to incorporate the process of allocating cognitive resources, i.e., attention, and implicitly interestingness, into the decision models. (I like to call this as "internal search", vs. external standard one, which can be regarded as cognitive process of decision maker at the intelligence activity stage a la H.A.Simon.)

Roughly speaking, concavity in probability weight affects RDEU optimizers to have tendency to delay their decisive timing of choice. Relation between concavity / convexity in probability and preference of delay resolution / early resolution has been observed by researchers who are dealing with distinct models with or without rank-dependent representation (Karni and Safra, 1990; Nerhring, 1999; Grant et al., 1998). Because of monotone convex / concave and inverse S shaped weighting functions can represent only with 1 or 2 attention peaks as for probability, and RDEU cumulate in accordance with rank of outcomes by its standard utility, I guess that they could have expand the idea by Dempster-Shafer 's theory, assuming the redistribution of probability mass in belief system represents it, in the iterative multi-choice test with uncertain knowledge base.

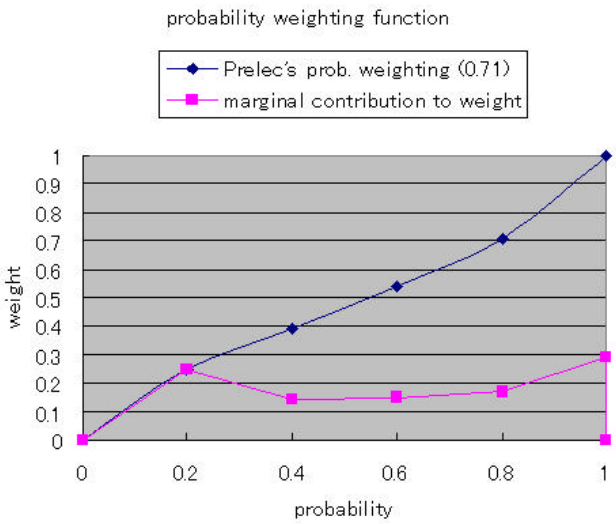


Figure 2.2 Prelec's probability weighting function Exp(-(-Ln p)^a) when a=0.71

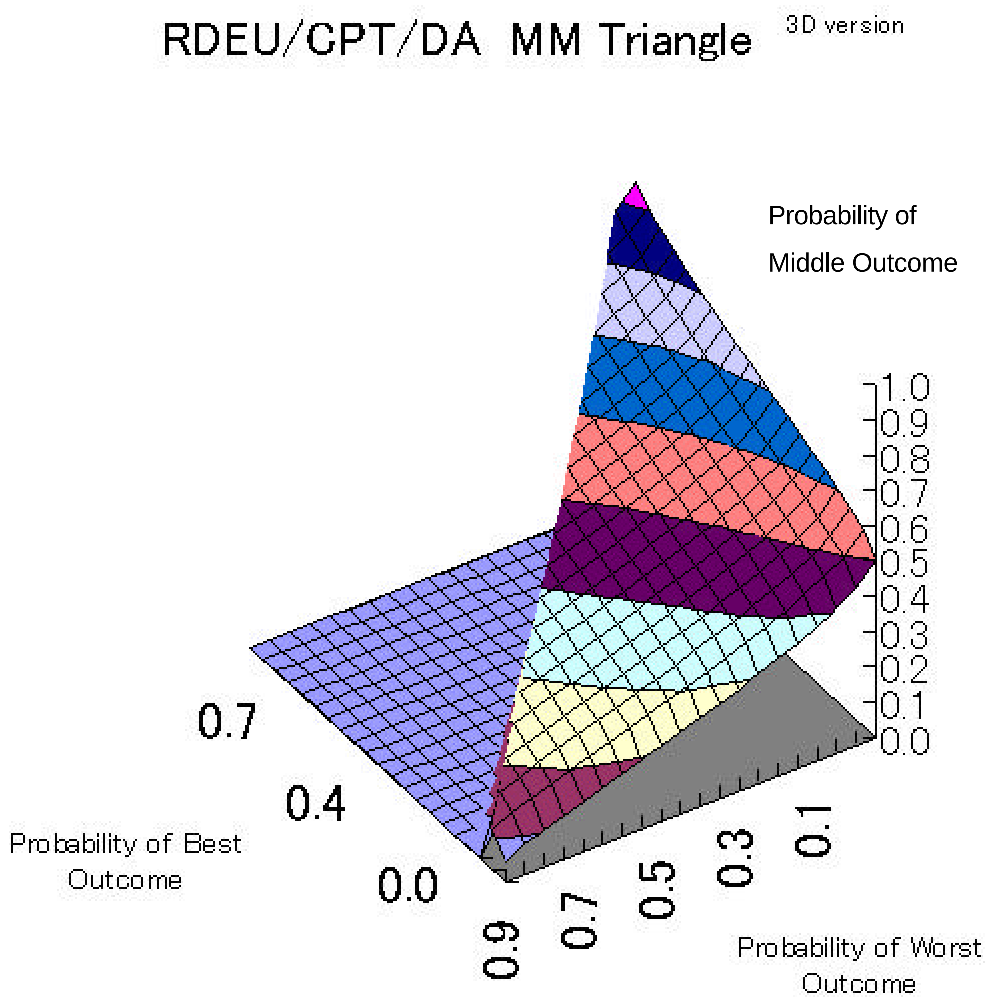


Figure 2.3 3-D visualization of the Marschack-Machina triangle for cumulative representations (RDEU with linear utility and decision weight Exp(-(-Ln p)^0.7).)

3. Review of cumulative representations and belief functions

The following is definitions and standard results about CEU (RDEU) and belief function reviewed briefly and rather informally. Readers with knowledge about these models can skip to next section.

Choquet Expected Utility Let $x_1 \geqq \cdots \geqq x_n$, $A_0 = \phi$、$v(\phi) = 0$.

$$CEU(f) = \Sigma_{k \in N} \left(v\left(\cup_{j<k} A_j \right) - v\left(\cup_{j<k} A_j \right) \right) \cdot u_k .$$

$$= \Sigma_{k \in N} v\left(\cup_{j<k} A_j \right) \cdot \left(u_k - u_{k+1} \right), \quad u_{n+1} = 0.$$

Usually, CEU models assume supermodularity (2-monotone convex capacity) and 0-1 normalization as for $v()$. And using its conjugate $v^*(E)=1-v(\neg E)$, a submodular function, to replace it in CEU definition formula is called Dual CEU (DCEU). RDEU representation is very similar to CEU with known probabilities prob (A_j), $j=1, \cdots, n$. In this case, we can regard $v()$ in above CEU representation as nonlinear (cumulative) probability weighting function.

$$CEU = \sum_{k\in N} \left(v^*\left(\cup_{j>k} A_j \right) - v^*\left(\cup_{j>k} A_j \right) \right) \cdot u_k$$

$$DCEU = \sum_{k\in N} \left(v^*\left(\cup_{j<k} A_j \right) - v^*\left(\cup_{j<k} A_j \right) \right) \cdot u_k$$

example. $n=2, x\geqq y$, $\Omega=\{A,B\}$.

$CEU=v(A)\,u(x)+(v(A\cup B)-v(A))\,u(y)$

$\quad =v(A)\,u(x)+(1-v(A))\,u(y)$

$v(A)\geqq v(\phi)$, $(X=A,\ Y=\phi,\ X\cup Y=A,\ X\cap Y=\phi)$

$v(B)\geqq v(\phi)$, $(X=B,\ Y=\phi,\ X\cup Y=B,\ X\cap Y=\phi)$

$v(A\cup B)+v(\phi)\geqq v(A)+v(B)$, $(X=A,\ Y=B,\ X\cup Y=A\cup B,\ X\cap Y=\phi)$

$v(\phi)=0$, $v(A\cup B)=1$

$v(A)=\alpha$, $v(B)=\beta \rightarrow \alpha+\beta\leqq 1$, $\alpha, \beta\geqq 0$.

ambiguity aversion: $c(v, A)=v(A\cup B)-v(A)-v(B)=1-\alpha-\beta=c(v, B)$.

CEU maximization coincides with Min of EU maximization with additive probability on the pessimistic expansion of original state space (Gilboa and Schmeidler, 1994; Mukerji, 1997; Gihiradato and Le Breton, 2000). Conjugate result of this generalizes classical result of Indirect Expected Utility with recursive representation of Kreps (Nehring, 1999).

example. A frame of discernment (Ω', Θ), and two act f and g. Let $f(\theta=a)=f(a)=5$, $f(\theta=b)=f(b)=1$, $g(\theta=a)=g(a)=2$, $g(\theta=b)=g(b)=3$. Then

$CEU(f)=[f(a)-f(b)]\cdot v(\{a\})+f(b)\cdot v(\{a, b\})$

$\quad =4v(\{a\})+v(\{a, b\})=4\alpha+1$,

$CEU(g)=[g(b)-g(a)]\cdot v(\{b\})+g(a)\cdot v(\{a, b\})$

$\quad =v(\{b\})+2v(\{a, b\})=\beta+2$.

Let $\Omega'=2^{\Theta}=\{\omega_0=\text{“}\phi\text{”}, \omega_1=\text{“}A\text{”}, \omega_2=\text{“}B\text{”}, \omega_3=\text{“}A\cup B\text{”}\}$. If we re-evaluate act on Ω' $\phi(\cdot\,;f)$ as its worst value of EU under additive probability measure p: $\Omega'\rightarrow[0, 1]$, then $\phi(\omega_1; f)=5$, $\phi(\omega_{j\neq 1}; f)=\min\{5, 1\}=1$. Since $p(\omega_0)=m(\phi)=0$, $p(\omega_1)=m(A)=\alpha$, $p(\omega_2)=m(B)=\beta$、

$p(\omega_3) = m(A \cup B) = 1 - \alpha - \beta$,

$EU(\phi(\cdot\,; f)) = \Sigma p(\omega_j)\,\phi(\omega_j\,; f)$

$= 5\alpha + \Sigma p(\omega_{j \neq 1})$

$= 5\alpha + (1 - \alpha)$

$= 4\alpha + 1 = CEU(f)$,

Similarly,

$EU(\phi(\cdot\,; g)) = \Sigma p(\omega_j)\,\phi(\omega_j\,; g)$

$= 3\beta + 2\Sigma p(\omega_{j \neq 2})$

$= 3\beta + 2(1 - \beta)$

$= \beta + 2 = CEU(g)$.

Generally, the following theorem is well known in literature (Gilboa and Schmeidler, 1994).

$$v(A) = \Sigma_{B \in \Sigma - \{\phi\}}\ m(B) \cdot e_A(B),\quad \Sigma = 2^{\Theta}.$$

$$\int f\, dv = \Sigma_{B \in \Sigma}\ m(B)\,[\min_{\omega \in B} f(\omega)]\ . \qquad \text{(Mean of Min)}$$

$$\int f\, dv = \min_{p \in Core(v)} \Sigma_{\omega \in \Omega}\ p(\{\omega\})\, f(\omega). \qquad \text{(Min of Mean)}$$

where unit capacity $e_A : 2^{\Omega} \to \{0, 1\}$, $e_A(B) = 1$ for $A \subseteq B$, $e_A(B) = 0$ otherwise.

As well as belief functions, the capacity about the CEU is one way to handle knowledge about the imprecise probability or imprecise knowledge of the agent (Dekel, Lipman and Rustichini, 1998). And it is insisted in literature, that the preference of decision makers who maximize CEU and Max-Min utility simultaneously, his /her nonadditive belief can be updated by Dempster-Shafer rule without any theoretical trouble (Gilboa and Schmeidler, 1993).

$$m_{DS}(A \mid B) = \Sigma_{E \cap B = A}\ m(E) \,/\, (1 - \Sigma_{E \cap B = \phi}\ m(E))\ .$$

Dempster-Shafer conditioning rule, or DS updating rule and Upper/Lower probabilities conditioning rule, comparing to Bayes rule, are stated as bellow (Moral and De Campos, 1991;Walliser, 1993; Dubois and Prade, 1997).

$$v_{DS}(A|B) = \frac{v(A \cup \neg B) - v(\neg B)}{1 - v(\neg B)} = \frac{v^*(B) - v^*(\neg A \cap B)}{v^*(B)}. \qquad \text{(DS)}$$

$$v_{UL}(A|B) = \frac{v(A \cap B)}{v(A \cap B) + v^*(\neg A \cap B)}. \qquad \text{(UL)}$$

$$p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{p(A \cap B)}{p(A \cap B) + p(\neg A \cap B)}. \qquad \text{(Bayes)}$$

Belief function in Dempster-Shafer's evidential reasoning model is defined as 0-1 normalized totally monotone capacity (Walley, 1991).

$$v\ (\cup_{i\in N} A_i)\ \geqq\ \Sigma_{J\subseteq N}\ (-1)^{\#J+1}\cdot v\ (\cap_{i\in J} A_i).$$

In finite case, basic probability assignment (b.p.a.) function $m()$, or mass in short, defines belief function.

$$v\ (A)\ =\ \Sigma_{B\subseteq A}\ m\ (B). \qquad \text{(belief function)}$$

An event with positive mass $m\ (B) > 0$, $B\subseteq\Theta$ is called focal element. Thus belief function is the sum of the mass of focal elements as its total evidential weight. Regarding this, it also called credibility measure, and its conjugate function is called plausibility function (or plausibility measure).

$$v^*\ (A)\ = 1 - v(\neg A) = \ \Sigma_{A\cap B\neq\phi}\ m\ (B) \qquad \text{(plausibility function)}$$

Mobius inversion (or inversion) defines b.p.a. based on belief function. It is easy to verify that inversion can be applied to capacities of above CEU example.

$$m\ (B)\ =\ \Sigma_{A\subseteq B}\ (-1)^{\#(B-A)}\ v\ (A) \qquad \text{(inversion)}$$

where $B-A=B\cap\neg A$, $m(\phi)=0$, $\Sigma_{A\subseteq\Omega}\ m(A)=1$.

DS rule has drawbacks in conflicting evidences (Murphy, 2000; Peral, 1991). If the all positive masses are only assigned to the singletons $E_k=\{\omega_k\}$, then (additive) it is a probability measure. The case of nested focal elements, $E_1\subseteq\cdots\subseteq E_n$, $E_k=\{\omega_1, \cdots, \omega_k\}$, $k=1, \cdots, n$ is called consonant belief function, and is same as Zadeh's fuzzy measure (Dubois and Prade, 1988).

$$v\ (A\cap B)\ =\ \min\ (v\ (A),\ v\ (B)). \qquad \text{(necessity measure)}$$

$$v^*\ (A\cup B)\ =\ \max\ (v^*\ (A),\ v^*\ (B)). \qquad \text{(possibility measure)}$$

And this will be inverted by the weight $\Sigma_{\omega\in A} m\ (A)$, such that $\pi:\Omega\rightarrow[0, 1]$, $1=\pi(\omega_1)\geqq\cdots\geqq\pi(\omega_n)\geqq\pi(\omega_{n+1})=0$.

$$v\ (A)\ =\ \min\ \{1-\pi(\omega),\ \omega\in\neg A\},$$

$$v^*\ (A)\ =\ \max\ \{\pi(\omega),\ \omega\in A\},$$

$$m\ (A_i)\ =\ \pi\ (\omega_i)-\pi\ (\omega_{i+1}), \qquad i=1,\ \cdots,\ n.$$

Nested focal elements has its relation to qualitative possibilities and epistemic entrenchment (Dubois and Prade, 1991; Wong et al., 1991; Walliser, 1993; Mongin, 1994)

4. Iterative multi-choice test: A web-based examination system

The iterative multiple-choice test is a web-based experimental examination system where the answer of students can be partially marked (partial judge, or pJudge) with self-allotment of points within endowment (for each question, 10 points, initially). This exam system is accessible from anywhere that has a connection to WWW of Internet (URL http://www.us.kanto-gakuen.ac.jp/kindo/). Programming language is JavaScript and html. CGI (Common Gateway Interface) has used minimally so that students can submit the experimental data by the exam system. The experimental data submitted by students include their answers, allotments, judged results, score, questionnaires for both of their subjective report of difficulty and performance about each question, timing of choice for selected options and partial judgment, and some other questionnaires about student 's objective attributes and opinions. Following part of this section has several figures of the exam system written in Japanese only, because of convenience for Japanese native students. Readers may skip to next section summary results.

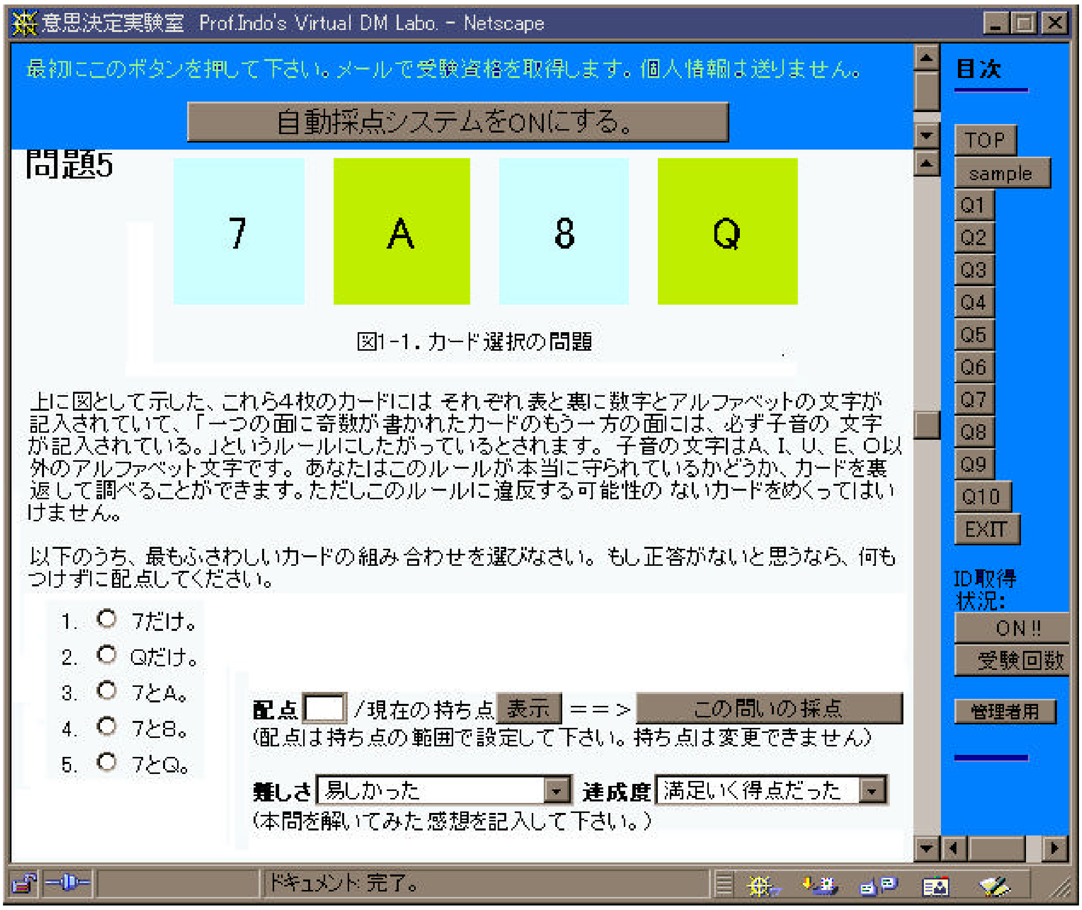


Figure 4.1 Problem Q4

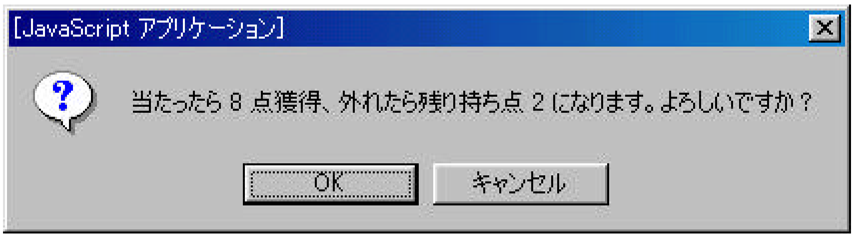


Figure 4.2 Confirm window to pJudge

The following is the explanation about this exam system in Japanese.

ＷＥＢ上に作成した実験システムの採点ルーチンは JavaScript でプログラムした（他の試験システムも兼ねているため、千ステップ程度ある)。この新しい多肢選択型試験では、受験者自身が持ち点のうち、自分の選んだ選択肢に対して自由に配点することが許される。採点も自動化されている。また外した場合は、持ち点から配点を減じた残りが０とならない限り、再チャレンジできる。それゆえ知識がなくても、盲目探索によって正解に達することはできるが、不正解のとき、配点分持ち点が減少するので回答者は正答見込みや自身の知識についての自信に応じた採点・配点の戦略をとることになる。選択肢のチェックは各問につき一度に１項目(１オプション)のみできる。このため部分的採点を繰り返すことにより、焦点の１つと残りという対比を繰り返したネスト（あるいはリスト）型情報構造が得られる。これはｋ回目迄の正解を焦点要素とした Consonant belief function（第３節）に対応する。

配点政策は自分自身でサーチの成功報酬を定義することであり、それゆえサーチ問題をデザインして自分自身にエージェントとして解かせる処方的役割付きの多重自己モデルの一例になっている。また配点を使い果たすことで、回答者はサーチ停止をコントロールできるが、このとき、見込みランキングに依存した果断性が、配点とその後の採点・配点行動に反映されると考えられる。例えば、最も見込みが高い選択肢が仮に１番だったとすると、これに持ち点すべてかけるのが（主観的）期待得点が最も高いが、知識ベースが果断ではないのにそうすることは事前には不安、そして事後には後悔ないし失望を生じさせるため、ある程度これらの心理を抑制する行動が予想できる。また不安や後悔は知識ベースの果断性が低くなるとその増加が逓減すると予想できる。後悔については各設問回答後にプルダウンメニューから記入してもらうアンケート方式を工夫した。またこの試験では、最初の説明において、選択肢の中に正答のない可能性が明記されており、無チェックで配点すれば正解になるという採点ルールが約束されている。しかしラジオボタンのオプションチェックは一度つけると、変更はできるが、ユーザー自身では無チェック状態には戻せないようになっている。これは熟慮に欠ける選択に対してペナルティとなるが、注意深くない被験者はこの可能性に気づいていないことがある。

5. Experimental data and simulation results

This section provides the summary results of our experimentation of iterative multi-choice test with 5 branches (5-taku in Japanese) where students are permitted to allocate endowments (i.e., 10 points, initially) to bet for his / her choice.

The experimental data to be analyzed are of 2 (+1) class, total 24 students ' submitted results (ID 8-31). 8 cases (ID 8--15) of Experiment A are time-controlled in my class about 1 hour. ID 16 and 17 are also categorized in A since both monitored but ended within 10 minutes. Remains 14 cases (ID 18-31) are of Experiment B under free-submit condition via Internet. 8 cases previously collected (ID 1-8) are excluded because it was same exam system but almost different problems. (Tables 5.1?-5.5, Figures 5.1--5.5)

And we show also some simulation results of simple 2-choice test (2-taku in Japanese) assuming RDEU /CPT-maximization (Figure 5.6). Then the elicitation-approximation procedure will be demonstrated by RDEU / CPT models. (In the case of CPT, we assumed linear utility.) (Figures 5.7-? 5.11) Firstly to determine a cumulative probability weight, with its decisiveness measure that the smallest penalty weight so that under which the observed sequential allocation pattern is optimal given the cumulative probability weight. And secondly, to find the approximation of this cumulative probability weight with consonant belief function and its conjugate plausibility function. Then probability masses represent a solution for decision maker 's (decentralized) cognitive resource allocation (i.e., attention) problem.

Table 5.1 Total time consumptions in experiment A (time controlled) and B (free submit)

time consumed

	average	SD	Max	Min
experiment A	0:43:46	0:07:55	0:56:12	0:35:18
experiment B	0:29:34	0:36:07	2:26:40	0:03:43

Table 5.2 Partial judges and the improvement (except for two objective questionnaires)

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total
%right@1st	71%	–	–	33%	50%	38%	25%	13%	42%	42%	39%
%right@fin	88%	–	–	58%	75%	75%	58%	71%	67%	75%	71%
improved	17%	–	–	25%	25%	38%	33%	58%	25%	33%	32%
mean #pJudge	1.3	1.0	1.0	1.7	1.4	1.4	1.8	1.4	1.4	1.7	1.4

Table 5.3 Reported subjective difficulty

	difficult_4	difficult_5	difficult_6	difficult_7	difficult_8	difficult_9	difficult_10	total_diff
#A	6	4	8	8	11	10	8	0
#B	4	6	2	4	1	5	0	2
#C	7	8	11	9	8	6	13	20
#D	7	6	3	3	4	3	3	2
#E	0	0	0	0	0	0	0	0
SUM	24	24	24	24	24	24	24	24
	difficult_4	difficult_5	difficult_6	difficult_7	difficult_8	difficult_9	difficult_10	total_diff
#A%	25%	17%	33%	33%	46%	42%	33%	0%
#B%	17%	25%	8%	17%	4%	21%	0%	8%
#C%	29%	33%	46%	38%	33%	25%	54%	83%
#D%	29%	25%	13%	13%	17%	13%	13%	8%
#E%	0%	0%	0%	0%	0%	0%	0%	0%
SUM	100%	100%	100%	100%	100%	100%	100%	100%

Table 5.4 Reported subjective performance

	achiev_4	achiev_5	achiev_6	achiev_7	achiev_8	achiev_9	achiev_10	satisfaction
#A	10	10	12	12	14	14	13	12
#B	6	7	7	3	2	6	4	12
#C	3	3	1	4	6	2	3	0
#D	5	3	4	4	2	1	4	0
#E	0	1	0	1	0	1	0	0
SUM	24	24	24	24	24	24	24	24
	achiev_4	achiev_5	achiev_6	achiev_7	achiev_8	achiev_9	achiev_10	satisfaction
#A%	42%	42%	50%	50%	58%	58%	54%	50%
#B%	25%	29%	29%	13%	8%	25%	17%	50%
#C%	13%	13%	4%	17%	25%	8%	13%	0%
#D%	21%	13%	17%	17%	8%	4%	17%	0%
#E%	0%	4%	0%	4%	0%	4%	0%	0%
SUM	100%	100%	100%	100%	100%	100%	100%	100%

Table 5.5 Total of partial judges for each questions and students

Number of p-judges for each question				cells colored are larger			between	subjects	within	subject	both		
student	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Average	SD	MAX	total
1	3	1	5	1	0	0	0	0	0	0	1.0	1.7	5	10
2	2	5	0	0	0	0	0	0	0	0	0.7	1.6	5	7
3	6	0	0	0	0	0	0	0	0	0	0.6	1.9	6	6
4	0	1	2	2	2	2	5	0	0	0	1.4	1.6	5	14
5	0	0	0	0	0	0	0	0	0	0	0.0	0.0	0	0
6	0	0	0	0	0	0	0	0	0	0	0.0	0.0	0	0
7	1	5	7	2	1	1	1	1	1	1	2.1	2.1	7	21
8	3	1	1	3	1	1	1	1	1	3	1.6	1.0	3	16
9	2	1	1	1	1	1	3	3	3	2	1.8	0.9	3	18
10	1	1	1	4	1	1	2	3	2	1	1.7	1.1	4	17
11	2	1	1	4	4	4	4	1	1	1	2.3	1.5	4	23
12	3	1	1	2	1	1	3	2	1	2	1.7	0.8	3	17
13	1	1	1	2	3	3	2	2	1	2	1.8	0.8	3	18
14	1	1	1	5	2	2	3	2	2	3	2.2	1.2	5	22
15	1	1	1	1	1	1	1	1	1	1	1.0	0.0	1	10
16	1	1	1	1	1	1	1	1	1	1	1.0	0.0	1	10
17	1	1	1	1	1	1	1	1	1	1	1.0	0.0	1	10
18	1	1	1	1	1	1	1	1	1	1	1.0	0.0	1	10
19	1	1	1	2	1	1	2	1	1	1	1.2	0.4	2	12
20	1	1	1	1	1	1	1	2	1	1	1.1	0.3	2	11
21	1	1	1	1	1	1	2	1	4	4	1.7	1.3	4	17
22	1	1	1	2	2	2	3	1	1	3	1.7	0.8	3	17
23	1	1	1	1	2	2	3	2	3	3	1.9	0.9	3	19
24	1	1	1	1	1	1	1	1	1	1	1.0	0.0	1	10
25	1	1	1	1	1	1	3	2	1	3	1.5	0.8	3	15
26	1	1	1	1	1	1	1	1	1	1	1.0	0.0	1	10
27	1	1	1	1	1	1	1	1	1	1	1.0	0.0	1	10
28	1	1	1	1	2	2	1	1	1	1	1.2	0.4	2	12
29	1	1	1	1	1	1	1	1	1	1	1.0	0.0	1	10
30	1	1	1	1	1	1	1	1	1	1	1.0	0.0	1	10
31	1	1	1	1	1	1	1	1	1	1	1.0	0.0	1	10
total	30	24	24	40	33	33	43	34	33	40	33.4		7	332
Average	1.3	1.0	1.0	1.7	1.4	1.4	1.5	1.4	1.4	1.7	1.4			
SD	0.6	0.0	0.0	1.2	0.8	0.8	1.0	0.7	0.8	1.0	0.7			
MAX	3	1	1	5	4	4	4	3	4	4	5			
(expect for 1-7)														

Table 5.6 Time consumptions for each questions and students

Total time consumptions				cells colored are larger			between	subjects	within	subject	both	
student	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Average	SD
1	0:02:17	0:00:40	0:01:04	0:00:28	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:27	0:00:45
2	0:12:31	0:04:25	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:01:42	0:04:03
3	0:08:26	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:51	0:02:40
4	0:00:00	0:00:23	0:00:21	0:00:23	0:00:24	0:00:21	0:00:36	0:00:00	0:00:00	0:00:00	0:00:15	0:00:13
5	0:01:18	0:01:05	0:00:11	0:00:47	0:00:31	0:00:15	0:00:11	0:00:12	0:00:15	0:00:13	0:00:30	0:00:25
6	0:01:21	0:01:30	0:00:06	0:00:04	0:00:11	0:00:00	0:00:15	0:00:00	0:00:00	0:00:00	0:00:21	0:00:35
7	0:05:33	0:01:53	0:01:31	0:00:36	0:00:42	0:00:26	0:00:29	0:00:24	0:00:21	0:00:25	0:01:14	0:01:36
8	0:02:36	0:07:00	0:01:27	0:06:32	0:04:19	0:08:31	0:06:20	0:03:19	0:04:01	0:05:17	0:05:15	0:02:37
9	0:02:06	0:04:44	0:00:39	0:03:43	0:01:57	0:04:29	0:06:06	0:03:34	0:03:20	0:04:53	0:03:33	0:01:37
10	0:06:45	0:03:40	0:02:39	0:08:45	0:04:45	0:08:14	0:04:22	0:04:16	0:04:32	0:01:39	0:04:52	0:02:14
11	0:01:45	0:03:56	0:01:06	0:03:42	0:02:52	0:07:36	0:03:46	0:03:15	0:03:06	0:04:39	0:03:35	0:01:45
12	0:01:47	0:04:36	0:01:34	0:01:52	0:02:23	0:02:55	0:02:37	0:01:47	0:02:10	0:01:36	0:02:20	0:00:55
13	0:00:59	0:02:12	0:02:49	0:03:19	0:02:01	0:02:47	0:02:43	0:02:26	0:01:34	0:04:07	0:02:30	0:00:53
14	0:03:54	0:03:44	0:01:11	0:04:12	0:02:51	0:02:09	0:03:06	0:01:25	0:02:13	0:04:19	0:02:54	0:01:06
15	0:00:41	0:00:38	0:00:11	0:00:31	0:02:07	0:04:11	0:00:37	0:00:15	0:00:13	0:00:12	0:00:58	0:01:16
16	0:00:20	0:00:25	0:00:32	0:00:14	0:00:30	0:00:34	0:00:26	0:00:20	0:00:40	0:00:18	0:00:26	0:00:08
17	0:00:21	0:00:37	0:00:23	0:00:07	0:01:38	0:00:08	0:00:18	0:00:21	0:00:34	0:00:29	0:00:30	0:00:26
18	0:00:52	0:00:17	0:00:13	0:00:08	0:00:17	0:00:36	0:00:53	0:00:15	0:00:23	0:15:01	0:01:54	0:04:37
19	0:14:56	0:05:20	0:04:21	0:22:20	0:24:31	0:25:42	0:11:49	0:18:04	0:11:58	0:07:39	0:14:40	0:07:48
20	0:05:44	0:00:33	0:00:33	0:00:56	0:00:18	0:00:21	0:01:29	0:00:39	0:00:29	0:05:02	0:01:36	0:02:02
21	0:01:18	0:01:07	0:00:11	0:00:24	0:03:06	0:05:41	0:04:46	0:00:43	0:02:27	0:03:21	0:02:18	0:01:54
22	0:00:14	0:00:13	0:00:15	0:00:20	0:00:24	0:00:36	0:00:33	0:00:23	0:00:11	0:00:32	0:00:22	0:00:09
23	0:03:03	0:00:44	0:00:39	0:02:12	0:03:13	0:05:26	0:02:17	0:04:16	0:02:09	0:02:14	0:02:37	0:01:26
24	0:01:54	0:00:46	0:01:33	0:01:07	0:02:19	0:01:17	0:00:18	0:00:13	0:00:12	0:00:14	0:00:59	0:00:46
25	0:00:52	0:02:36	0:00:51	0:02:33	0:01:46	0:02:45	0:02:23	0:01:45	0:01:07	0:02:53	0:01:57	0:00:47
26	0:01:13	0:01:12	0:01:36	0:00:28	0:00:26	0:00:33	0:00:25	0:00:25	0:00:24	0:00:29	0:00:43	0:00:27
27	0:00:33	0:00:11	0:00:14	0:00:11	0:00:10	0:00:00	0:00:15	0:00:15	0:00:26	0:00:35	0:00:17	0:00:11
28	0:00:18	0:00:20	0:00:10	0:00:13	0:00:31	0:00:14	0:02:06	0:00:17	0:00:13	0:00:14	0:00:28	0:00:35
29	0:03:51	0:00:52	0:00:30	0:01:06	0:00:29	0:00:35	0:01:13	0:00:43	0:00:43	0:00:36	0:01:04	0:01:01
30	0:01:38	0:01:05	0:00:23	0:00:58	0:00:19	0:00:42	0:01:25	0:00:46	0:01:34	0:00:44	0:00:57	0:00:28
31	0:07:43	0:02:18	0:01:10	0:00:52	0:00:43	0:01:18	0:00:46	0:00:25	0:00:34	0:00:26	0:01:37	0:02:13
total	1:04:23	0:48:14	0:25:12	1:08:49	1:03:56	1:28:20	1:00:52	0:50:07	0:45:13	1:07:29	0:58:21	
Average	0:02:41	0:02:03	0:01:03	0:02:52	0:02:40	0:03:41	0:02:32	0:02:05	0:01:53	0:02:49	0:02:26	
SD	0:03:16	0:01:58	0:01:01	0:04:47	0:04:51	0:05:27	0:02:41	0:03:40	0:02:30	0:03:22	0:03:21	
(expect for 1-7)												

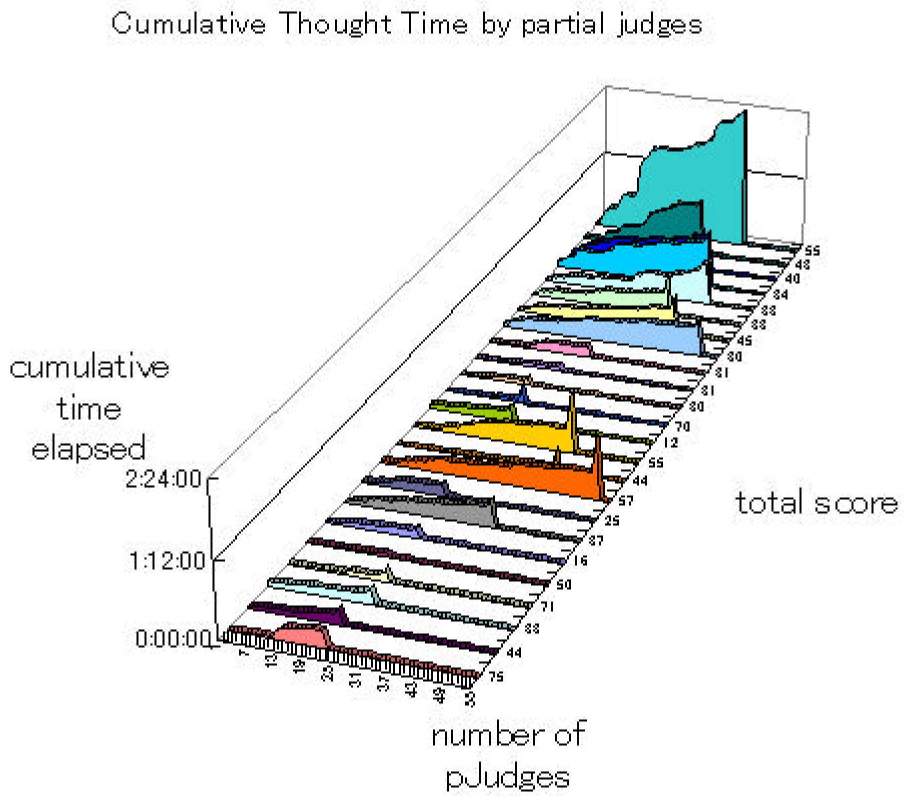
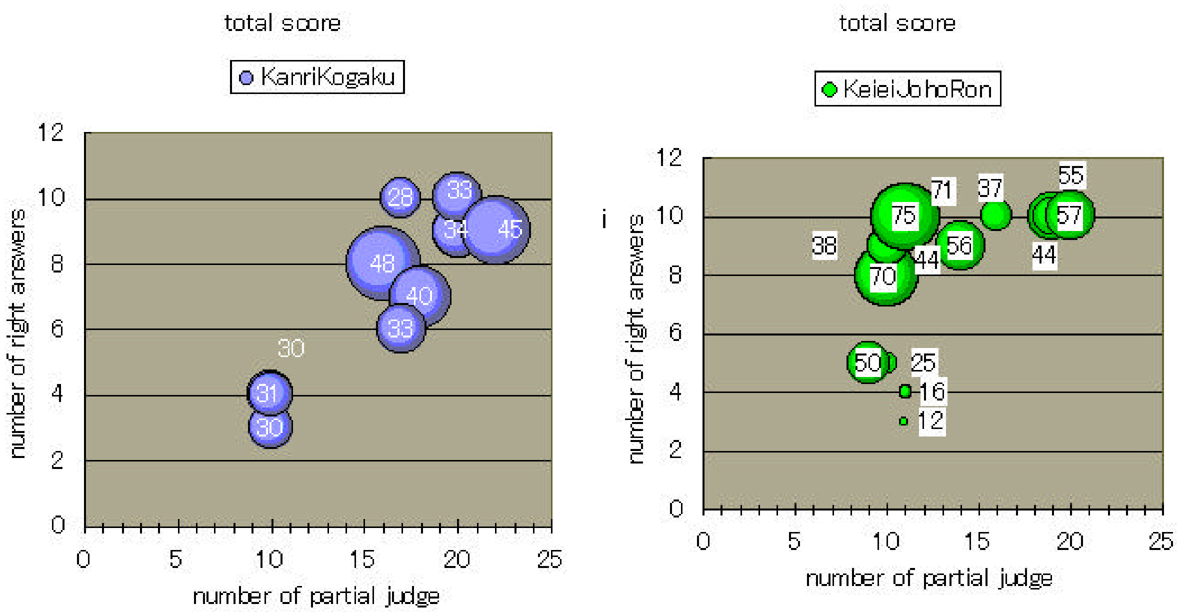


Figure 5.1. Cumulative time consumptions over the trials of partial judgment



(a)Experiment A: controlled total time (b)Experiment B: free submit condition

Figure 5.2 Total score

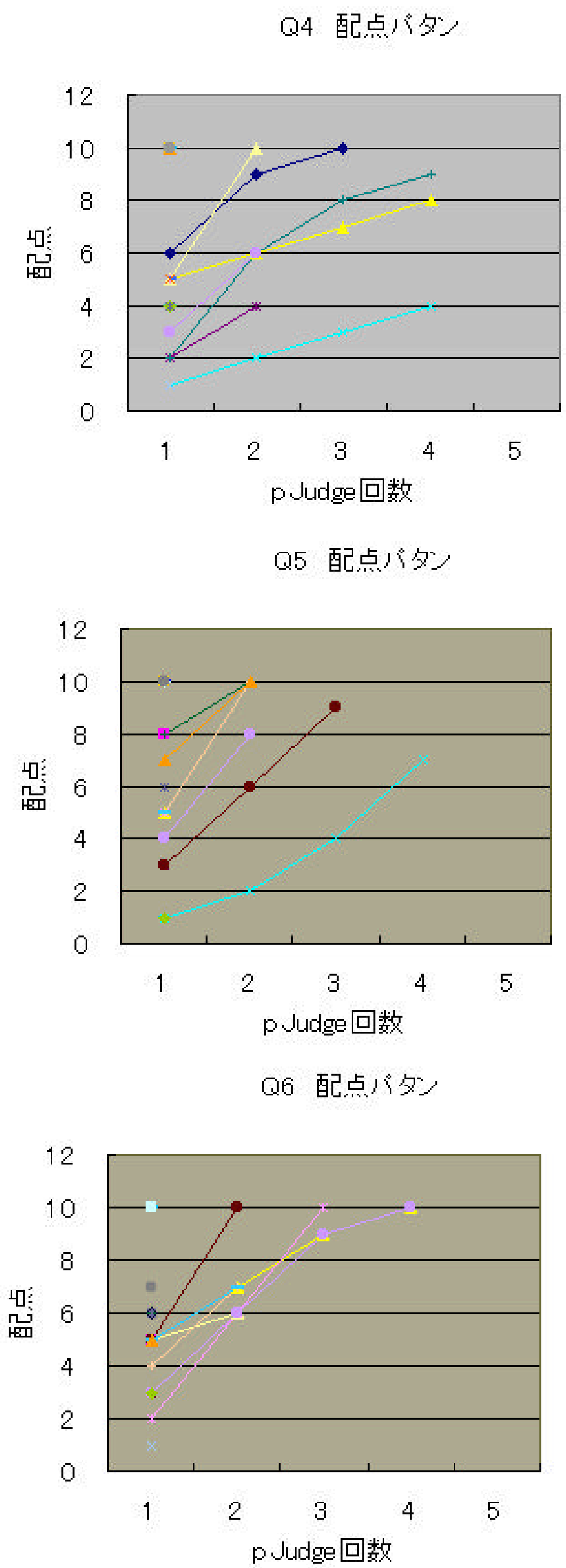


Figure 5.3 Cumulative allocation patterns (Q4-6)

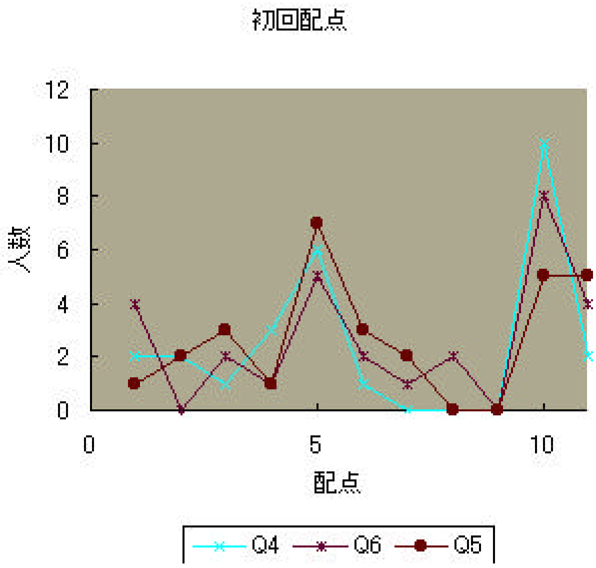
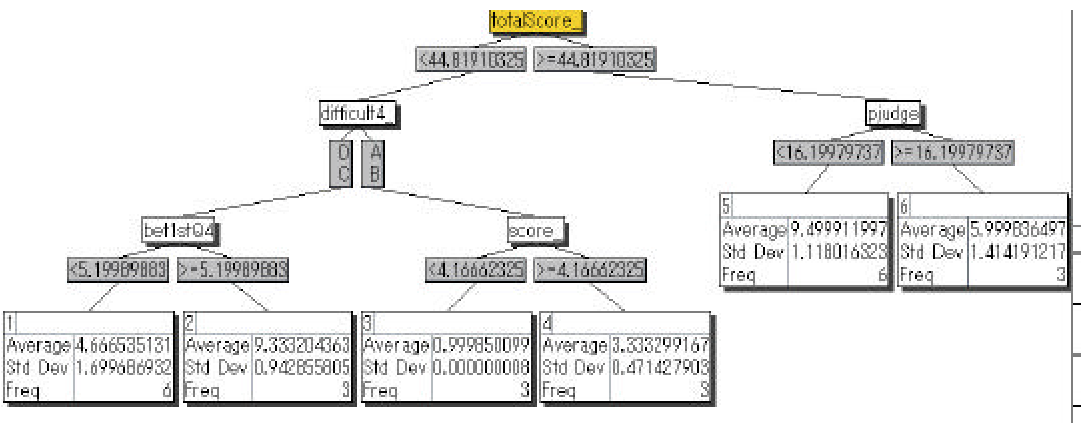


Figure 5.4 Initial allocation pattern in experimental data (Q4～6)



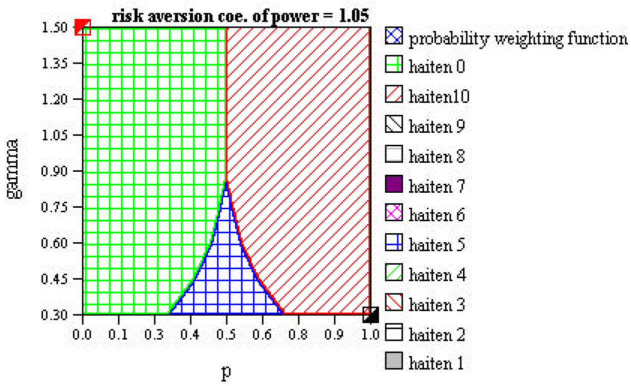
(5.5a) The result of tree induction by using Attar's XR Profiler

```
<Layer Level 1>
  Rules To Reach Leaf Profile 1
       IF  difficult4_ =  C
       THEN bet1stQ5 = 5   (Probability = 0.5714)
  Rules To Reach Leaf Profile 2
       IF  difficult4_ =  A OR  D OR  B
       THEN bet1stQ5 = 10   (Probability = 0.3750)
<Layer Level 2>
  Rules To Reach Leaf Profile 1
       IF  achiev4_ =  D
       AND difficult4_ =  C
       THEN bet1stQ5 = 8   (Probability = 1.0000)
  Rules To Reach Leaf Profile 2
       IF  achiev4_ =  A OR  B OR  C
```

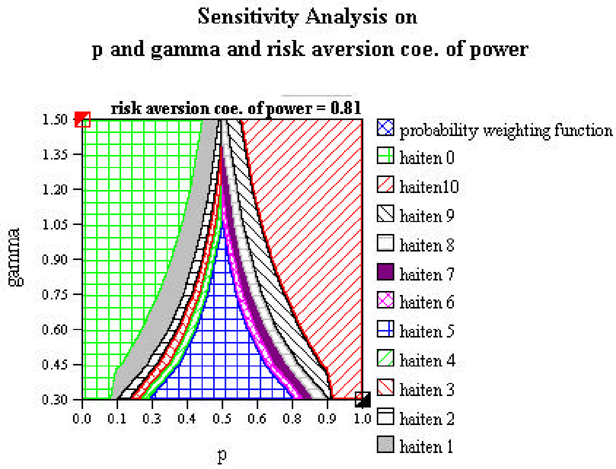
```
        AND difficult4_ =  C
        THEN bet1stQ5 = 5   (Probability = 0.6667)
   Rules To Reach Leaf Profile 3
        IF  pjudge < 17
        AND difficult4_ =  A OR  D OR  B
        THEN bet1stQ5 = 10   (Probability = 0.5455)
   Rules To Reach Leaf Profile 4
        IF  pjudge >= 17
        AND difficult4_ =  A OR  D OR  B
        THEN bet1stQ5 = 3   (Probability = 0.4000)
```

(5.5b) the result of rule induction using same software above

Figure 5.5 The induced uncertain knowledge base as for 1st partial judge in Q5.



(5.6a) power utility function (1.05)



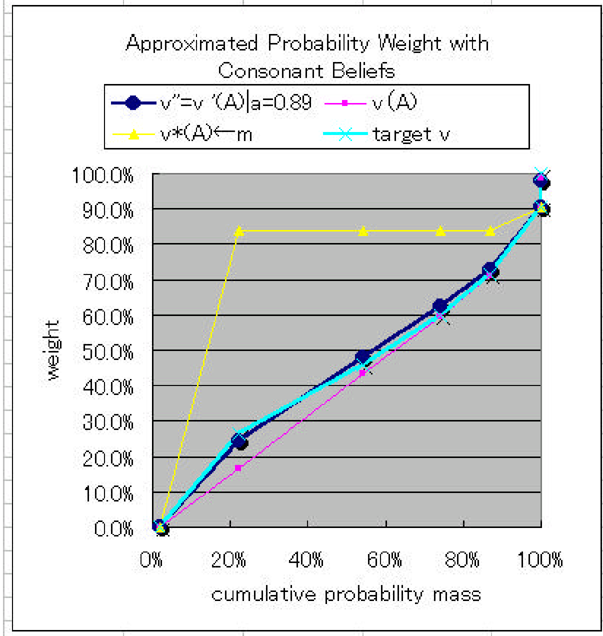
(5.6b) power utility function (0.81)

Figure 5.6 A sensitivity analysis of 2-choice test simulation of CPT-maximization with Prelec's weighting function and power utility function (using TreeAge's DATA 3.5)

#	A_1	A_2	A_3	A_4	A_5	Meaning	Count	v(A)	m(A)	v*(A)←
0	0	0	0	0	0	最後まで正解しない。	1	0%	0%	0%
1	1	0	0	0	0	1回目に正解する	2	17%	17%	84%
2	1	1	0	0	0	2回目までに正解する	1	43%	27%	84%
3	1	1	1	0	0	3回目までに正解する	1	60%	16%	84%
4	1	1	1	1	0	4回目までに正解する	2	71%	12%	84%
5	1	1	1	1	1	5回目までに正解する	2	91%	13%	91%
6	1	1	1	1	1	*まだ始めていない。	2	99%	0%	91%
7	1	0	0	0	0	*1が正解であった。	2	90%	0%	84%
8	0	1	0	0	0	2が正解であった。	1	0%	0%	67%
9	0	0	1	0	0	3が正解であった。	1	0%	0%	40%
10	0	0	0	1	0	4が正解であった。	1	0%	0%	24%
11	0	0	0	0	1	5が正解であった。	2	7%	7%	19%
12	0	1	1	1	1	1は正解でなかった。	2	7%	0%	74%
13	0	0	1	1	1	2までは正解でなかった	1	7%	0%	47%
14	0	0	0	1	1	3までは正解でなかった	1	7%	0%	31%
15	0	0	0	0	1	*4までは正解でなかっ7	2	12%	0%	19%
16	0	1	1	1	1	*1は不正解である。	2	54%	0%	74%
17	1	0	1	1	1	2は不正解である。	1	23%	0%	91%
18	1	1	0	1	1	3は不正解である。	1	50%	0%	91%
19	1	1	1	0	1	4は不正解である。	1	66%	0%	91%
20	1	1	1	1	0	*5は不正解である。	2	54%	0%	84%
21	1	0	1	0	0	21	1	17%	0%	84%
22	1	0	0	1	0	22	1	17%	0%	84%
23	1	0	0	0	1	23	1	23%	0%	91%
24	1	1	0	1	0	24	1	43%	0%	84%
25	1	1	0	0	1	25	1	50%	0%	91%
26	1	0	1	1	0	26	1	17%	0%	84%
27	1	0	1	0	1	27	1	23%	0%	91%
28	1	0	0	1	1	28	1	23%	0%	91%
29	0	1	1	0	0	29	1	0%	0%	67%
30	0	1	0	1	0	30	1	0%	0%	67%
31	0	1	0	0	1	31	1	7%	0%	74%
32	0	1	1	0	1	32	1	7%	0%	74%
33	0	1	1	1	0	33	1	0%	0%	67%
34	0	0	1	1	0	34	1	0%	0%	40%
35	0	0	1	0	1	35	1	7%	0%	47%
36	0	1	0	1	1	36	1	7%	0%	74%
	19	19	19	19	19			0%	**91%**	

(5.7a) An example of consonant (2-monotone convex) belief approximation of inverse S shaped nonlinear weighting function and its marginal contributions (using Excel + Solver + Evolver), in accordance with the search order (not the ranking of outcomes). The convexity | concavity dispersions from additive probability measure of this consonant approximation for that weight (that is the square sum of the sum of dispersions in each column and row) are 1068.079918 | 9.74121E-18 respectively. And the basic probability masses are computed by Mobius inversion, which is the technique in game theory to compute agent 's contribution against given coalition.

Approximating Probability Weights				**Total_error**	75.2			
v''=v '(A)|a=0.89				*10000	**a**			
m_cumulative	Target m	**m_Error**	Target v	**v_error**	Predict m	Predict v	0.8881496	
		59.0686	convex_0.7	16.1354	m(A)	v(A)	**v '(A)**	**m '(A)**
2%	2%	-2.0%	0%	0.0%	0.0%	0.0%	0.0%	0.0%
22%	20%	-3.3%	26%	2.2%	16.7%	16.7%	24.2%	24.2%
54%	32%	-5.3%	46%	-1.8%	26.7%	43.4%	47.9%	16.2%
74%	20%	-3.7%	60%	-2.5%	16.2%	59.6%	62.3%	18.9%
87%	13%	-1.5%	71%	-1.2%	11.5%	71.1%	72.6%	7.5%
100%	13%	-0.4%	91%	0.0%	12.6%	90.5%	90.5%	12.6%
100%	0%	0	100%		0.0%	98.9%	98.0%	0.0%
	100%				0.0%	0.0%	9.4%	0.0%
					0.0%	0.0%	7.5%	7.5%
					0.0%	0.0%	4.5%	4.5%
					0.0%	0.0%	2.7%	2.7%
					6.8%	6.8%	8.2%	8.2%
					0.0%	6.8%	14.3%	-1.4%
					0.0%	6.8%	11.3%	1.4%
					0.0%	6.8%	9.5%	-1.4%
					0.0%	2.2%	4.1%	0.0%
					0.0%	8.3%	15.6%	0.0%
					0.0%	23.4%	30.9%	-1.4%
					0.0%	50.1%	54.6%	-1.4%
					0.0%	66.4%	69.1%	-1.4%
					0.0%	9.4%	17.7%	0.0%
					0.0%	16.7%	24.2%	-4.5%
					0.0%	16.7%	24.2%	-2.7%
					0.0%	23.4%	30.9%	-1.4%
					0.0%	43.4%	47.9%	2.7%
					0.0%	50.1%	54.6%	1.4%
					0.0%	16.7%	24.2%	2.7%
					0.0%	23.4%	30.9%	1.4%
					0.0%	23.4%	30.9%	1.4%
					0.0%	0.0%	7.5%	-4.5%
					0.0%	0.0%	7.5%	-2.7%
					0.0%	6.8%	14.3%	-1.4%
					0.0%	6.8%	14.3%	1.4%
					0.0%	0.0%	7.5%	2.7%
					0.0%	0.0%	4.5%	-2.7%
					0.0%	6.8%	11.3%	-1.4%
					0.0%	6.8%	14.3%	1.4%
shift graph of v ' verticaly			0		**90.5%**			**90.5%**



(5.7b) The approximated inverse S shaped weight by linear combination of the consonant convex capacity found in Figure 5.7a and its conjugate concave function with pessimistic ratio 0.888, with total error 75.2. We observed that at the small sacrifice of the exactness of above consonant belief, total error less than 45 was obtained. When S shaped, the inversed mass is not a bpa because of its nonmonotonicity, so that there exist several negative values. However, I think that the out of consonant components of this approximation are considered to being distributed partial knowledge "on the spots" of decision makers which will activated when the spot information corresponding to each event becomes accessible dynamically.

Figure 5.7 An example of consonant belief approximation of weighting function by simulation

options	A_1	A_2	A_3	A_4	A_5	A_0	0誤差限界		
trial (p.Judge) order	1	2	3	4	5	NA	SUM	residual	
prob	20%	20%	20%	20%	20%	0%	1	0	0
cumulative	20%	40%	60%	80%	100%	100%	p_error*		0
Prelec's prob weighting (0.71)	25%	39%	54%	71%	100%	100%	0.00		
marginal contribution to weight	25%	14%	15%	17%	29%	0%			
conditional probabilities	20%	25%	33%	50%	100%	52%	-		0-tolerance
Ranking	1	1	1	1	1	6	SUM	residual	1E-15
allotment (haiten)	1.833	1.833	1.833	1.833	1.833	0.833	10.0	-1E-06	
inequality	-0.167	-0.333	-0.500	-0.667	-0.833	ieq=	-2.5		penalty weight
experimental data to fit	2	4	2	1	1	0	x_error		for x_error
	MAX	MIN	Range	EV	target param		6.8333		-0.0005
	10	0	10	1.833334	gamma of	0.71			
power utility (normalized)	0.18	0.18	0.18	0.18	0.18	0.08		upper	27
expected_utility (power)	0.04	0.04	0.04	0.04	0.04	0.00	0.18	lower	40
choquet integral (RDEU/CPT)	0.05	0.03	0.03	0.03	0.05	0.00	0.18	L-U	13
							-		↓
weighting function	2	<== 1: convex/concave capacity, 2 or otherwise: Prelec function							Min.
convexity of power weighting	0.9962		Evolver		alpha	beta	gamma	gap_U	0.096
optimistic coefficient (beta)	0.12		Best値		0.7	0.7	0.7	upper	0.186027
risk averse index (alpha)	1.00		Criteria->	EV	EU	Hurwicz	RDEU	lower	0.292276
gamma of Prelec's prob weight	0.71	Penalty=	[illegible]	3.20	0.41	0.70	0.38		
target cell (maximization)	0.18675	1	EV(比較)	3.20	3.20	3.50	3.25	simulation parameter	
input your utility functional #	4	RDEU	EU(比較)	0.41	0.41	0.35	0.41	number of intervals	
1:EV, 2:EU, 3:Hurwicz, 4:RDEU, 5:Anxiety			Hurwicz(同	0.21	0.21	0.70	0.55		40
icz 配点最適化			RDEU(同)	0.38	0.38	0.36	0.38	max penalty weight	
U			(current)	1.83	0.18	0.18	0.18		0.02
ety									
	MAX	MIN	Range	Hurwicz	EU	RDEU/OPT	Anxiety		
	0.2	0.2	0.0	0.18	0.18	0.18	0.19		

Figure 5.8 A snapshot of the spreadsheet simulation for eliciting nonlinear probability weighting. In this case, the best probability such that it minimizes the upper bound of the penalty weight to attain optimality of given pattern of allocation is even at the level of 20% among 5 options and thus any pattern of allocation optimizes the cumulative utility. Note that in this case the meaning of closedness of a pattern to the best, usually a decisive one, has transposed. Since any decisive (or indecisive) pattern of allocation as well as a flat allocation, can be rationalized by equal probabilities and near to linear weight over probabilities and by any nonlinear weight with even marginal contributions to cumulative probabilities, this deficiency of our elicitation method may be handled by consideration of some sort of preference for variation. But it is adding of ad hoc constraints without justification.

ID	weight	haltenf	halt	halt	halt	halt	halt	target	EV	EU	Hurwicz	RDEU	Anxiety	model	penalty	p_err	x_error
1	0.0195	2	4	2	1	1	0	0.1848	2	0.2	0.12671	0.187	0.2101	4	0.00224	0	0.1151
2	0.019	2	4	2	1	1	0	0.1849	2	0.2	0.12649	0.1872	0.21009	4	0.0023	0	0.12124
3	0.0185	2	4	2	1	1	0	0.1849	2	0.2	0.12626	0.1873	0.21008	4	0.00237	0	0.12788
4	0.018	2	4	2	1	1	0	0.185	2	0.2	0.12601	0.1874	0.21007	4	0.00243	0	0.13506
5	0.0175	2	4	2	1	1	0	0.1851	2	0.2	0.12575	0.1876	0.21006	4	0.0025	0	0.14291
6	0.017	2	4	2	1	1	0	0.1851	2	0.2	0.12548	0.1877	0.21004	4	0.00257	0	0.15144
7	0.0165	2	4	2	1	1	0	0.1852	2	0.2	0.12519	0.1879	0.21003	4	0.00265	0	0.16076
8	0.016	2	4	2	1	1	0	0.1853	2	0.2	0.12488	0.188	0.21002	4	0.00274	0	0.17096
9	0.0155	2	4	2	1	1	0	0.1854	2	0.2	0.12456	0.1882	0.21	4	0.00282	0	0.18217
10	0.015	2	4	2	1	1	0	0.185	2	0.2	0.124	0.188	0.210	4	0.0029	0	0.19
11	0.0145	2	4	2	1	1	0	0.186	2	0.2	0.124	0.189	0.210	4	0.0030	0	0.21
12	0.014	2	4	2	1	1	0	0.186	2	0.2	0.123	0.189	0.210	4	0.0031	0	0.22
13	0.0135	2	4	2	1	1	0	0.186	2	0.2	0.123	0.189	0.210	4	0.0032	0	0.24
14	0.013	2	4	2	1	1	0	0.186	2	0.2	0.123	0.189	0.210	4	0.0034	0	0.26
15	0.0125	2	4	2	1	1	0	0.186	2	0.2	0.122	0.190	0.210	4	0.0035	0	0.28
16	0.012	2	4	2	1	1	0	0.186	2	0.2	0.121	0.190	0.210	4	0.0036	0	0.30
17	0.0115	2	4	2	1	1	0	0.186	2	0.2	0.121	0.190	0.210	4	0.0038	0	0.33
18	0.011	2	4	2	1	1	0	0.187	2	0.2	0.120	0.191	0.210	4	0.0040	0	0.36
19	0.0105	2	4	2	1	1	0	0.187	2	0.2	0.120	0.191	0.210	4	0.0042	0	0.40
20	0.01	2	4	2	1	1	0	0.187	2	0.2	0.119	0.191	0.210	4	0.0044	0	0.44
21	0.0095	2	4	2	1	1	0	0.187	2	0.2	0.118	0.192	0.210	4	0.0046	0	0.48
22	0.009	2	4	2	1	2	0	0.187	2	0.2	0.117	0.192	0.210	4	0.0049	0	0.54
23	0.0085	2	4	2	1	2	0	0.198	2	0.2	0.116	0.193	0.210	4	0.0051	0	0.61
24	0.008	2	4	2	1	2	0	0.188	2	0.2	0.115	0.194	0.210	4	0.0055	0	0.68
25	0.0075	2	4	2	1	2	0	0.188	2	0.2	0.113	0.194	0.210	4	0.0058	0	0.78
26	0.007	2	4	2	1	2	0	0.189	2	0.2	0.112	0.195	0.209	4	0.0063	0	0.89
27	0.0065	2	4	2	1	2	0	0.189	2	0.2	0.110	0.196	0.209	4	0.0067	0	1.04
28	0.006	2	4	2	1	2	0	0.190	2	0.2	0.108	0.197	0.209	4	0.0073	0	1.22
29	0.0055	2	3	2	1	2	0	0.191	2	0.2	0.105	0.198	0.209	4	0.0080	0	1.45
30	0.005	2	3	1	1	2	0	0.191	2	0.2	0.103	0.200	0.209	4	0.0088	0	1.75
31	0.0045	3	3	1	1	2	0	0.192	2	0.2	0.099	0.202	0.209	4	0.0097	0	2.16
32	0.004	3	3	1	1	2	0	0.194	2	0.2	0.094	0.204	0.209	4	0.0109	0	2.74
33	0.0035	3	3	1	1	2	0	0.195	2	0.2	0.089	0.208	0.208	4	0.0125	0	3.57
34	0.003	3	3	1	1	3	0	0.197	2	0.2	0.081	0.212	0.208	4	0.0146	0	4.86
35	0.0025	3	3	1	0	3	0	0.200	2	0.2	0.071	0.218	0.208	4	0.0175	0	7.00
36	0.002	3	3	1	0	3	0	0.204	2	0.2	0.062	0.226	0.207	4	0.0219	0	10.94
37	0.0015	4	2	0	0	4	0	0.212	2	0.2	0.049	0.241	0.206	4	0.0292	0	19.50
38	0.001	4	1	0	0	5	0	0.225	2	0.2	0.061	0.262	0.202	4	0.0367	0	36.72
39	0.0005	3	0	0	0	7	0	0.250	2	0.2	0.079	0.278	0.200	4	0.0280	0	58.02
40	-1E-17	0	0	0	0	10	0	0.292	2	0.2	0.117	0.292	0.200	4	0.0000	0	106.00

Figure 5.9 Counterfactual ranking and decisiveness measure. This is an output of spreadsheet simulation with SOLVER, the standard optimization tool of Excel (a product of Microsoft), iteratively executed by VBA macro I coded. In each iteration, the same target cell is REDU-like cumulative representation (but ranked by search order not by outcomes) given an Prelec-type inverse S shaped weight with index .71 (it is the setting in Figure 5.8) has maximized varying linear penalty weight w.r.t. "x_error" (i.e., the square sum of approximation error) given allocation pattern of points 2-4-2-1-1-0, a case in the experiments (case 14, Q4), with 40 intervals respectively. The counterfactual ranking of allocation patterns ranked by the lower bound of linear weight of penalty function, each of which is optimal given the cumulative probability weight. The decisiveness measure in this simulation is 13 = 40 ? 27 with tolerance 1.0 in x_error. (Optimized allocation patterns in above output are rounded.)

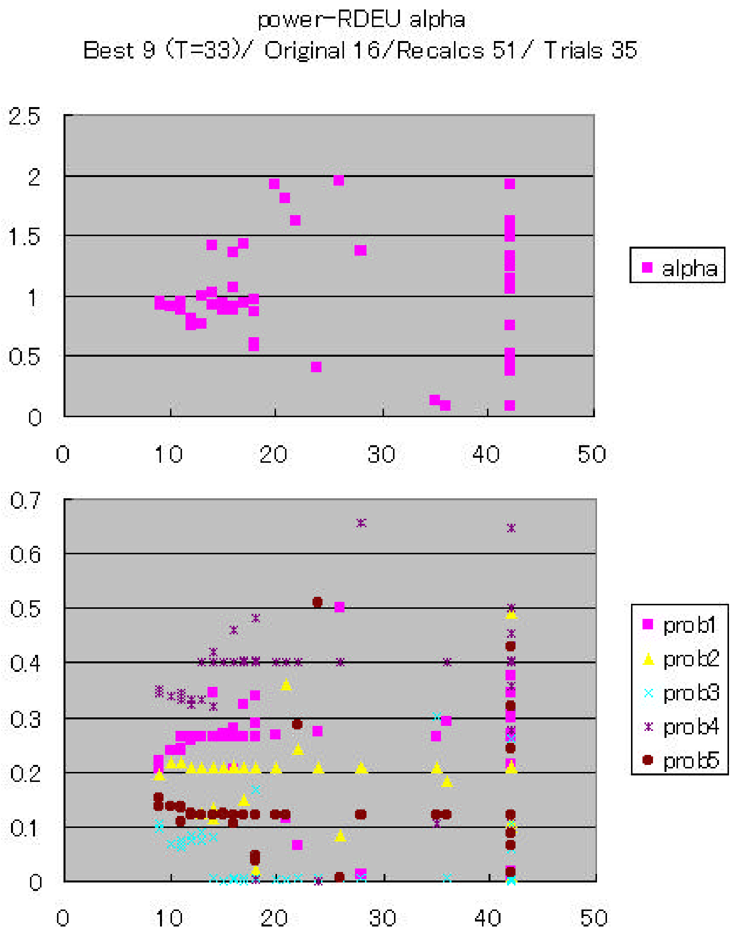


Figure 5.10 An output of simulation in the elicitation procedure of our method, using Evolver, GA optimization tool, Palisade 's product, addin tool for Excel, where Evolver iteratively execute the VBA macro in Figure 5.9 varying probabilities and (the parameter of) probability weighting functions so that it minimizes the decisiveness measure. The horizontal axis represents the minimum value of the penalty function weight in order to justify the allocation pattern given a probability weight.

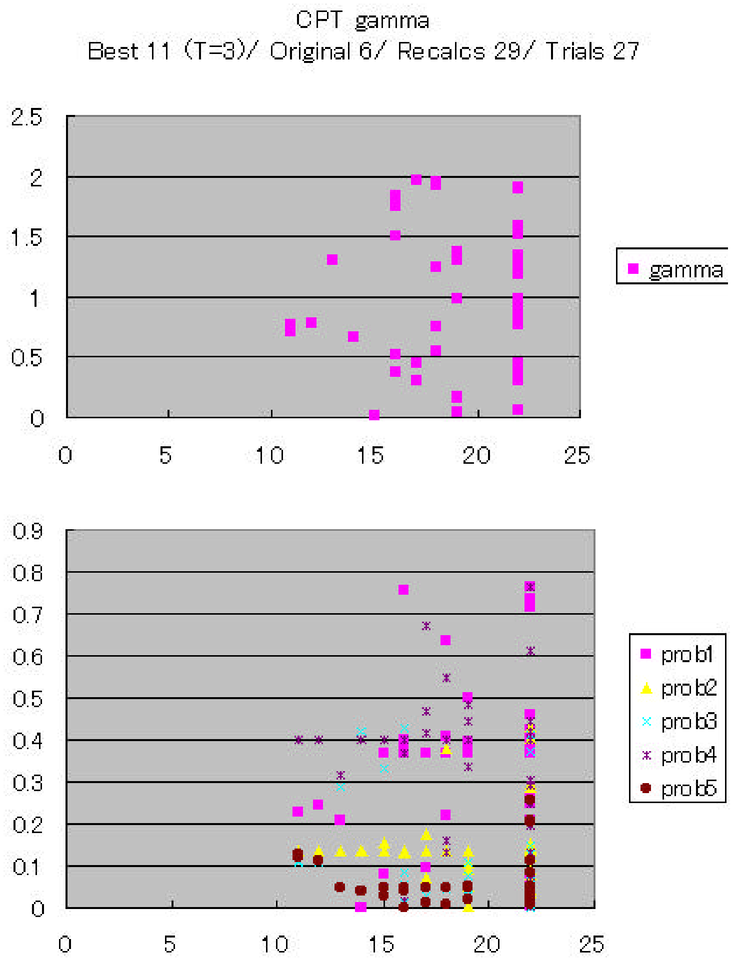


Figure 5.11 GA optimization for the elicitation of inverse S-shaped weight Exp(-(-Ln p)^gamma) assuming cumulative representation. The horizontal axis is same as Figure 5.10.

6. Conclusion

In this paper, based on cumulative representations, some preliminary experimentations using human subjects and computer simulation models of elicitation of distributed knowledge of decision makers in the iterative multi-choice test has demonstrated. I found the notion of decisiveness that it is based on counterfactual ranking of weighting functions, and consonant belief approximation for nonlinear probability weighting are both useful, but it seems that further investigation is needed.

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