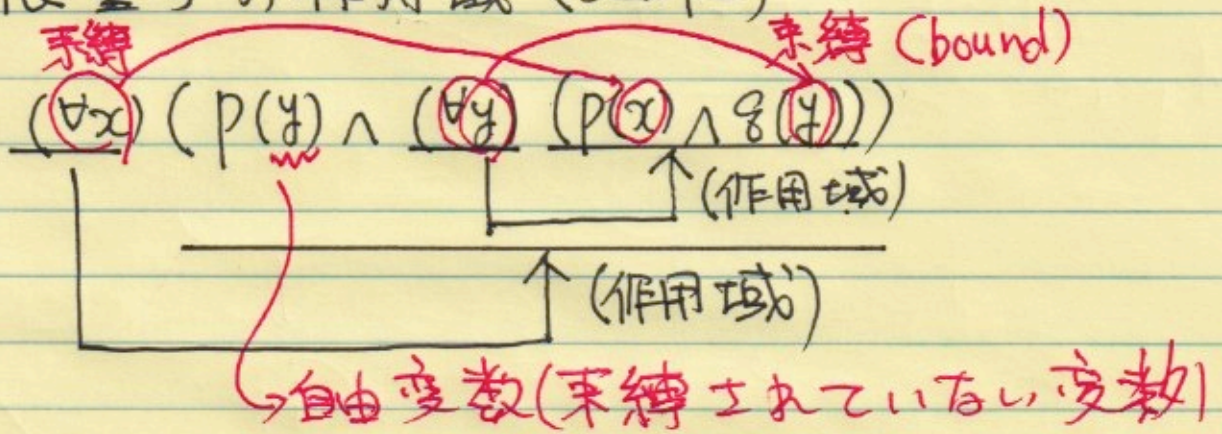
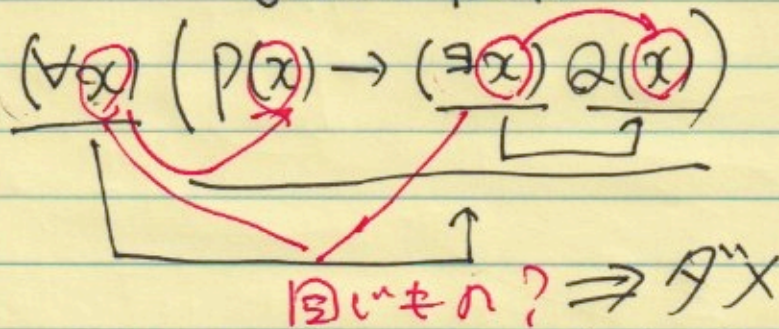


① 限量子の作用域 (scope)



$P(x) \rightarrow (\exists x) Q(x)$   
 $\Downarrow$  全称閉形



② ~~$(\exists x) P(x) \wedge (\exists x) Q(x) \neq (\exists x) (P(x) \wedge Q(x))$   
 $(\exists x) F(x) \wedge (\exists x) G(x) \neq (\exists x) (F(x) \wedge G(x))$~~
 ではない理由

(例)  $a \neq b \wedge \perp$  if  $x = a$   $\begin{cases} P(x) = F \\ \text{otherwise } G(x) = T \end{cases}$   
 if  $x = b$   $\begin{cases} P(x) = T \\ G(x) = F \end{cases}$   
 otherwise  $P(x) = G(x) = T$



②

$(\forall x) F(x) \vee (\forall x) G(x) \neq (\forall x) (F(x) \vee G(x))$  理由

(例)  $a \neq b$  とし

if  $x = a$  ならば  $\begin{cases} F(x) = \text{真} \\ G(x) = \text{偽} \end{cases}$

if  $x = b$  ならば  $\begin{cases} F(x) = \text{偽} \\ G(x) = \text{真} \end{cases}$

otherwise  $F(x) = G(x) = \text{偽}$

を考へよ。

$(\forall x) F(x) = \text{偽}$

$(\forall x) G(x) = \text{偽}$

よって  $(\forall x) G(x) \vee (\forall x) F(x) = \text{偽}$

$(\forall x) (F(x) \vee G(x)) = \text{真}$  □

演

①  $(\forall x) P(x) \rightarrow (\exists x) Q(x)$

$= \neg (\forall x) P(x) \vee (\exists x) Q(x)$

$= (\exists x) \neg P(x) \vee (\exists x) Q(x)$

$= (\exists x) (\neg P(x) \vee Q(x))$

②  $(\forall x) (\forall y) [(\exists z) (P(x, z) \vee P(y, z)) \rightarrow (\exists u) Q(x, y, u)]$

$= (\forall x) (\forall y) [\neg (\exists z) (P(x, z) \vee P(y, z)) \vee (\exists u) Q(x, y, u)]$

$= (\forall x) (\forall y) [(\forall z) (\neg P(x, z) \wedge \neg P(y, z)) \vee (\exists u) Q(x, y, u)]$

$= (\forall x) (\forall y) (\forall z) (\exists u) (\neg P(x, z) \wedge \neg P(y, z) \vee Q(x, y, u))$