

令和5年度 入学者選抜学力試験（前期）

数 学

注 意 事 項

1. 試験開始の合図があるまで，この問題冊子と解答冊子を開かないでください。
2. 問題は必須問題と選択問題に分かれています。
3. 必須問題は2問あり，それらは1ページにあります。選択問題は問題範囲ごとにそれぞれ2問ずつあります。数学Ⅰ・数学Ⅱ・数学A・数学Bの問題は2ページに，数学Ⅲの問題は3ページにあります。
4. 解答冊子は，必須問題用と選択問題用の2冊に分かれています。それぞれの解答冊子の表紙の所定欄に氏名と受験番号をはっきりと記入してください。
5. 選択問題は解答する問題範囲を選び，選択問題用解答冊子の表紙の解答問題欄の選択欄に○印を記入してください。○印を記入していない場合，または複数の選択欄に○印を記入した場合は，0点となります。
6. 計算用紙は，解答冊子の中にとじてあります。
7. 試験中に問題冊子や解答冊子の印刷不明瞭，ページの落丁・乱丁，汚れ等に気がついた場合は，静かに手を挙げて監督員に知らせてください。
8. 試験終了後，問題冊子は持ち帰ってください。
9. 解答時間は120分です。
10. 問題ごとに配点が記されています。

必須問題

- I** 大小2個のさいころを同時に投げるとき，出る目をそれぞれ a, b とする．このとき，座標平面上の直線 $ax + by = 1$ を ℓ とし， ℓ と x 軸の交点を A ， ℓ と y 軸の交点を B とする．さらに原点を O とし， $\triangle OAB$ の $\angle OAB$ を θ とする．以下の問いに答えよ．（配点 75 点）

問1 $\cos \theta < \frac{1}{3}$ となる確率を求めよ．

問2 $2 \sin \theta \cos \theta$ が整数となる確率を求めよ．

問3 $\triangle OAB$ の内接円の半径が有理数となる確率を求めよ．

- II** 座標平面上の曲線 $y = x|x - 2|$ を C とし，直線 $y = mx$ を ℓ とする．ただし， $0 < m < 2$ とする．また，曲線 C と直線 ℓ で囲まれた部分の面積を S とする．以下の問いに答えよ．（配点 75 点）

問1 曲線 C と直線 ℓ を同一の座標平面上に図示せよ．

問2 面積 S を m を用いて表せ．

問3 面積 S が最小となるときの m の値を求めよ．ただし，そのときの S の値を求める必要はない．

必須問題は，このページで終了である．

選択問題 (数学 I ・ 数学 II ・ 数学 A ・ 数学 B)

I n を自然数とする. 正の実数 $a = \sqrt[3]{2 + \sqrt{5}}$, $b = \sqrt[3]{-2 + \sqrt{5}}$ を用いて, 数列 $\{c_n\}$ を $c_n = a^n - b^n$ と定める. 以下の問いに答えよ. (配点 75 点)

問 1 ab, c_3, c_1 がそれぞれ整数であることを示せ.

問 2 $\frac{c_2}{\sqrt{5}}, \frac{c_4}{\sqrt{5}}$ がそれぞれ整数であることを示せ.

問 3 $f(a) = f(b) = 0$ となるような, 整数係数の 4 次多項式 $f(x)$ を 1 つ求めよ.
また, 漸化式 $c_{n+4} = Ac_{n+2} + Bc_n$ を満たすような定数 A および B の値をそれぞれ求めよ.

問 4 c_{2n-1} および $\frac{c_{2n}}{\sqrt{5}}$ がそれぞれ正の整数であることを示せ. さらに, c_{2n-1} が $(c_{2n})^2 - 1$ の約数であることを示せ.

II x の範囲を $0 < x < \frac{\pi}{2}$, a を正の定数とする. また, 次のように x に関する条件 p , q を定める.

$$\text{条件 } p : (x - a)^2 \left(x - \frac{1}{a} \right) \geq 0$$

$$\text{条件 } q : -1 < \log_{(\cos x)} \left(8 \cos^3 x - 8 \cos x + \frac{1}{\cos x} \right) < 0$$

以下の問いに答えよ. (配点 75 点)

問 1 $\cos 4x - \cos x$ を 2 つの三角関数の積の形に変形し, $\cos 4x \leq \cos x$ を満たす x の値の範囲を求めよ.

問 2 条件 p を満たす x の範囲を a を用いて表せ.

問 3 $\cos 4x$ を $\cos x$ を用いて表せ. また, 条件 q を満たす x の値の範囲を求めよ.

問 4 命題「 $q \Rightarrow p$ 」が, $0 < x < \frac{\pi}{2}$ のすべての x に対して成り立つような a の値の範囲を求めよ.

数学 I ・ 数学 II ・ 数学 A ・ 数学 B の問題は,
このページで終りである.

選択問題 (数学 III)

- I** 定数 p を 3 以上の奇数, i を虚数単位とし, $\alpha = \cos\left(\frac{2\pi}{p}\right) + i \sin\left(\frac{2\pi}{p}\right)$ とする. また, 複素数 z に対して $f(z) = \frac{1}{z+1}$ とし, 複素数 z の偏角 $\arg z$ を $0 \leq \arg z < 2\pi$ の範囲で考える. n を p 以下の自然数とすると, 以下の問いに答えよ.

(配点 75 点)

問 1 α^n の実部および虚部を, p と n を用いてそれぞれ表せ.

問 2 $f(\alpha^n)$ は実部 $\frac{1}{2}$ の複素数であることを示せ.

問 3 $\tan(\arg f(\alpha^n))$ の最大値とそのときの n を, p を用いてそれぞれ表せ.

- II** n を自然数とし, 定積分 I_n を

$$I_n = \int_1^n \frac{1}{x(x^2+1)} dx$$

と定める. 以下の問いに答えよ. (配点 75 点)

問 1 $x \geq 1$ のとき, $x+1 \leq x^2+1 \leq 2x^2$ であることを用いて, 次の不等式を証明せよ.

$$\frac{1}{4} \left(1 - \frac{1}{n^2}\right) \leq I_n \leq \log\left(\frac{2n}{n+1}\right)$$

問 2 $\frac{1}{x(x^2+1)} = \frac{a}{x} + \frac{bx+c}{x^2+1}$ が x についての恒等式となるように, 定数 a, b, c の値を定めよ. さらに, 定積分 I_n を求めよ.

問 3 極限 $\lim_{n \rightarrow \infty} I_n$ を求めよ.

数学 III の問題は, このページで終りである.

令和5年度 入学者選抜学力試験（前期）

外国語(英語)

注 意 事 項

1. 試験開始の合図があるまで、この問題冊子と解答冊子を開かないでください。
2. 問題は1ページから10ページにあります。
3. 解答冊子の表紙の所定欄に氏名と受験番号をはっきりと記入してください。
4. 下書き用紙は、解答冊子の中に綴じてあります。
5. 試験中に問題冊子や解答冊子の印刷不明瞭、ページの落丁・乱丁、汚れ等に気づいた場合は、静かに手を挙げて監督員に知らせてください。
6. 試験終了後、問題冊子は持ち帰ってください。
7. 解答時間は90分です。
8. Partごとに配点が記されています。
9. 英語辞書を使用することができます。

Part 1: Reading Comprehension I

(配点 80 点)

Use the information in the article to answer questions (1) – (8). For each question, choose one answer (A, B, C or D).

Sustainable aviation fuel taking off worldwide

The move toward sustainable aviation fuel (SAF) derived from cooking oil, household garbage and other materials is gaining momentum in the airline industry, which has been the target of criticism overseas because of the high carbon dioxide emissions associated with flying.

In late March, aircraft manufacturer Airbus SE flew an A380 jumbo jet for about three hours powered solely by SAF for a test flight in Toulouse in southwestern France, indicating the safety of SAF and signaling a wave of change in the aviation industry.

The term “flight shaming” was popularized by environmental activist Greta Thunberg. In 2019, the Swedish teenager crossed the Atlantic Ocean by yacht when she traveled to the U.N. headquarters in New York for a climate summit, instead of traveling by plane.

Jet fuel derived from crude oil is responsible for most of the carbon dioxide emissions produced by the airline industry, which has come under increased scrutiny amid a global push for decarbonization.

The sense of urgency is particularly strong in Europe, where environmental issues attract more attention. European countries have started setting goals for the introduction of SAFs, which currently account for less than 1% of the total global supply of aviation fuels.

In Norway, it has been mandatory for airlines to use SAF mixed with other fuels since 2020, and Britain wants 75% of aviation fuel to be powered by SAFs by 2050.

A shift has also been seen among Japanese airlines. In March, All Nippon Airways, Japan Airlines and 14 other companies established an organization called Act For Sky to promote the use of domestically produced SAFs.

The organization, whose members include Nissin Foods Holdings Co., Odakyu Electric Railway Co. and a manufacturer of fuels derived from cooking oil, is working to secure raw materials and establish distribution networks.

ANA and JAL, competitors in the aviation industry, have already joined hands to promote the use of SAFs. In June, both airlines flew regular flights using fuel mixed with sustainable aviation fuel derived from microalgae and wood chips.

The government wants 10% of the aviation fuel used by domestic airlines to be SAFs by 2030.

The International Civil Aviation Organization, a United Nations agency, aims to adopt this year a target of net-zero carbon dioxide emissions among international airliners by 2050. As a result, efforts by the world's airlines are likely to accelerate.

“European and North American airlines might cut trips to Japan if planes cannot refuel with SAFs at Japanese airports,” said a senior official of the Land, Infrastructure, Transport and Tourism Ministry.

Securing raw materials is one of many challenges that lie ahead.

Euglena Co. produces health foods made from a type of algae called euglena, and also manufactures fuel derived from the aquatic organism and used cooking oil.

Last year, Honda Aircraft Co. flew a HondaJet private plane using an SAF manufactured by Euglena, which plans to build a mass production site in the future so that it can supply fuel to major airlines.

[...]

Keeping costs down will be a challenge, too. SAFs are three to four times the price of conventional aviation fuels. ANA wants companies to shoulder part of the burden in return for certificates to use the fuel.

Source:

The Japan News (2022, April 12). Sustainable aviation fuel taking off worldwide. *The Yomiuri Shimbun*. Retrieved from <https://japannews.yomiuri.co.jp/science-nature/climate-change/20220412-19729/>

(1) What is the main idea of the article?

- (A) People should stop travelling by plane to reduce carbon dioxide emissions.
- (B) SAFs are less economical than jet fuel made from crude oil.
- (C) European countries require their airlines to use SAFs.
- (D) There are increased efforts to use SAFs.

(2) According to the article, why is the airline industry using SAFs?

- (A) to reduce the cost of flying
- (B) to reduce the environmental impact of flying
- (C) to reduce the production of household garbage
- (D) to improve the performance of aircraft engines

(3) According to the article, what did Airbus SE do in late March?

- (A) It manufactured an A380 aircraft.
- (B) It conducted a test flight.
- (C) It released a new type of SAF.
- (D) It transferred its headquarters to New York.

(4) According to the article, what did Greta Thunberg do in 2019?

- (A) She promoted the use of cooking oil and household garbage in the aviation industry.
- (B) She travelled to New York on a plane powered by SAF.
- (C) She encouraged people to fly on planes powered by SAFs.
- (D) She went to a climate summit at the U.N. headquarters.

(5) According to the article, what proportion of the worldwide supply of aviation fuels are SAFs?

- (A) less than 1%
- (B) 10%
- (C) 75%
- (D) More than 99%

(6) According to the article, why was Act For Sky established?

- (A) to encourage the use of Japanese-made SAFs
- (B) to research SAF manufacturing technology
- (C) to manufacture SAFs from microalgae
- (D) to increase competition within the aviation industry

(7) According to the article, from which one of the following can SAFs be made?

- (A) crude oil
- (B) hydrogen
- (C) wood chips
- (D) industrial waste

(8) According to the article, why is adopting SAFs difficult?

- (A) Governments have limited the amount of SAFs that can be made.
- (B) The process of manufacturing SAFs is dangerous.
- (C) Environmental activists oppose the use of SAFs.
- (D) SAFs are more expensive than conventional aviation fuels.

Part 2: Writing I

(配点 50 点)

In your opinion, what are some of the advantages and disadvantages of air travel? Write about 100 words in English.

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Part 3: Reading Comprehension II

(配点 40 点)

Use the information in the article to answer questions (1) – (4). For each question, choose one answer (A, B, C or D).

Care robot a 'night shift work friend' at Japanese nursing home

OTSU -- A self-navigating artificial intelligence-equipped robot is doing its bit as part of the night shift crew at a nursing home in this west Japan city, helping to improve care while assisting in infection countermeasures.

A robot with faint blue lights quietly opens the door of a room at the Activa Biwa nursing home in the city of Otsu just past 9 p.m., when the lights are turned off for residents. After making sure that the resident in need of care is in bed, the robot silently closes the door.

The Aeolus robot became part of the Activa Biwa staff in November 2021. Using UV-C lights, it disinfects commonly touched places in the building including handrails and doorknobs. It can also detect abnormalities in residents as they sleep based on their posture or movements, and notify human coworkers by sharing images it has taken of the residents.

Its sensors allow the robot to self-navigate around the building, avoiding obstacles. It can operate an elevator on its own using arms with seven joints each. It was even able to prevent one resident from falling after detecting them trying to get into their wheelchair from their bed.

It makes the rounds on one floor of the facility's care ward, which has about 30 rooms, in about 45 minutes. The nursing home operator plans to add more Aeolus robots and increase the number of floors they look after.

Kayo Kojima, 60, the central Japan area manager for Activa Biwa's operating firm Trust Garden, told the Mainichi Shimbun, "The nursing care business will be even more short-staffed as Japan's population ages. To secure good service qualities, we've introduced a robot now, and we want to expand the things we can do."

Care unit leader Yuta Minamisawa, 26, who often works with the Aeolus robot on night shifts, said of his robotic coworker, "I was worried at first that it might create more work, but it's helped reduce work by making the rounds and disinfecting appliances. It's become my night shift work friend that I can rely on."

Source:

Mainichi Japan (2022, February 28). Care robot a 'night shift work friend' at Japanese nursing home. *The Mainichi*. Retrieved from <https://mainichi.jp/english/articles/20220226/p2a/00m/0sc/019000c>

(1) According to the article, what does the robot at Ativa Biwa currently do?

- (A) It socializes with the residents.
- (B) It helps the residents change their clothes.
- (C) It provides the residents with medicine.
- (D) It sanitizes surfaces people frequently touch.

(2) According to the article, what did Kayo Kojima say about the nursing care business?

- (A) Care robots will become more skilled than human workers.
- (B) There will not be enough people to work in nursing homes.
- (C) Residents will prefer robots to humans.
- (D) The quality of nursing home staff will increase.

(3) According to the article, why does the robot at Ativa Biwa share images?

- (A) to promote friendship among residents of the nursing home
- (B) to let staff know of possible problems with residents
- (C) to inform staff of surfaces that need disinfecting
- (D) to show coworkers that it is doing its job

(4) According to the article, what is Trust Garden's plan for care robots?

- (A) increase the number of floors that robots monitor
- (B) improve the robots' ability to avoid obstacles
- (C) have the robots work during the day shift
- (D) add speech functionality to the robots

Part 4: Writing II

(配点 30 点)

If you were a resident in a nursing home, would you prefer a human or robot caregiver? Write about 60 words in English.