Towards Formalizing Jazz Piano Knowledge with a Deductive Object-Oriented Approach

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Abstract

The aim of this research is to provide a formalization method for the musical concepts and entities observed in scores and texts, that is, musical knowledge. As a first step, the knowledge domain of the research is restricted to a jazz piano solo performance, and the knowledge activities of a jazz pianist are represented using the deductive object-oriented (DOO) method. The advantages of the DOO method bring about that musical knowledge is systematized by a partial ordering, the flexible object representation makes it possible to manipulate musical knowledge incrementally, and the rules in a clausal form realizes useful and complicated functions, such as property inheritance, similarity and deductive inference.

First, this paper shows a sample problem that the transcription of an actual solo performance by a jazz pianist is presented, explained, and analyzed. We are motivated to formalize the various musical knowledge appearing in the sample problem. From the sample problem, the observation that the piano performance analysis consists of abstraction and association is obtained. In the DOO framework, the abstraction relation can be naturally represented by a partial order $\sqsubseteq$, and the association by connecting different musical concepts to each other.

Next, the DOO representation starts with a note, and the ordering of sets is discussed. To represent a chord, a new data structure, μ term, is introduced. Then, it is demonstrated that the abstraction in the sample problem is naturally translated to the $\sqsubseteq$-ordering.

Next, a method of forming associations by attributes is introduced. Moreover, a new notation, called a dotted notation, useful for designating attribute values is introduced. The examples of voice leading and chord structure are presented.

The author is prototyping a musical knowledge base system employing the formalization method presented in this paper. The prototype system is supposed to store actual performances, scores and their explanations as they are. Then, the music knowledge is managed along the $\sqsubseteq$-ordering. Thus, least abstraction upon storing avoids to lose necessary information. Last, sample queries to the system are shown.

1 Introduction

This paper presents a method of formalizing jazz piano knowledge, that is based on the deductive object-oriented approach.

In general, a musician needs to manipulate and organize various kinds of musical knowledge not only during performance but also in exercises and rehearsals. Such knowledge activities consist of storing, retrieving, searching, reasoning, clustering and unifying information. Those activities are research subjects in the fields of artificial intelligence (AI) and knowledge processing (KP) [1] [4]. Thus, many research results of AI and KP should be helpful in the formalizing of musician's activities. Furthermore the musician's activities as research subjects may yield new insights into AI and KP.

This research focusses on the knowledge activities of a jazz solo pianist, since the musical knowledge that a jazz pianist handles is familiar to the author. Toward the simulation of jazz pianist activities, first, we should develop a formal knowledge representation method for jazz piano solo performance. Of the many knowledge representation methods have been studied in AI and KP. This research adopted the deductive object-oriented (DOO) method [5] [3]. The advantages offered by the DOO method are that the relations among objects are considered as a partial ordering (in the mathematical sense), an object whose information is partially known can be naturally specified, and the expression of rules in a clausal form enables deductive inference.

As a starting point, the transcription of an actual solo performance by a jazz pianist is presented, explained, and analyzed [2]. Once the musical knowledge contained in these materials is extracted and represented using the DOO method, we can treat the knowledge in a formal way. That allows useful operations, such as associative retrieving, analogy, belief revision, nonstandard reasoning and inductive learning, to be easily designed and

implemented.

2 Sample Problem

Figure 1 shows the first two bars of a solo performance by a jazz pianist; the tune title is Autumn Leaves [2]. The pianist added the following comment to his performance:

> The voicing of C_{m7} does not belong to the conventional 3rd-build system [1]. It has three perfect 4th intervals ($C5-F5-B^\flat5-E^\flat6$). Some of the inside voices of C_{m7} move to the inside voices of the subsequent F_7 chord by a semitone up ($B^\flat5 \rightarrow B5$, $F5 \rightarrow G^\flat5$, $C5 \rightarrow D^\flat5$, $B^\flat4 \rightarrow B4$). The voicing of F_7 also has two 4th intervals ($D^\flat5-G^\flat5-B5$).

Figure 2 shows a supplementary score for this comment.

An apprentice analyzes the scores and the comment, studies the new musical concepts, and applies them in other situations later. How should these knowledge activities be formalized? What kind of representation method efficiently describes these knowledge activities? In the following, let us consider a knowledge representation method for jazz piano, using Figures 1 and 2 and the above comment as a sample problem.

We assume that the piano performance analysis consists of abstraction and association. An apprentice abstracts the actual performance in Figure 1, and finally reaches Figure 2 and the comment previously quoted. The comment tells us that there are various associations among musical concepts, e.g., a set of notes is associated with a chord name. Next, it is noted that the abstraction from Figure 1 to Figure 2 includes three kinds of abstraction: grouping, removing and relativizing. In the first bar of Figure 1, the $E^\flat6$ note and the $B^\flat5$ and $F5$ notes have different onset times, but in Figure 2, they make a group and are considered as a single chord conceptually. Although three 8th notes of the opening melody $G5A5B^\flat5$ occur in Figure 1, they are removed in Figure 2 because they are not so important from the musical structure point of view. In Figure 1, the onset time and the duration of each note are specified; the timing information is determined absolutely. However, in Figure 2, the timing information is abstracted and only the relative position of each note is specified.

The following sections formalize this observation.

3 Abstraction

This section formalizes the abstraction process in the analysis of jazz piano performance based on the DOO method.

3.1 Note

For instance, consider how to represent a note whose pitch-class is C, octave 5, onset time 36 and duration 24. Here we think of it as a note object with four attributes (p, o, t, d for short). Thus, a note is represented as $n(p = C, o = 5, t = 36, d = 24)$. The argument positions in this notation are arbitrary. Next, suppose that there is a note object such that only the values of its pitch-class and octave attributes are known. Following our notation, we write $n(p = C, o = 5)$. Since the number of attributes of $n(p = C, o = 5)$ is less than that of $n(p = C, o = 5, t = 36, d = 24)$, we may derive that $n(p = C, o = 5)$ is more abstract than $n(p = C, o = 5, t = 36, d = 24)$; inversely $n(p = C, o = 5, t = 36, d = 24)$ is more instantiated than $n(p = C, o = 5)$. This ordering is denoted as

$$n(p = C, o = 5, t = 36, d = 24) \sqsubseteq n(p = C, o = 5).$$

Here the partial order $\sqsubseteq$ is called a subsumption relation and is regarded as an extension to the *is_a* relation [5].

For instance, consider a more abstract note object $n(p = C)$, which does not state its octave attribute. So we may interpret that any value can be assigned to the octave attribute. Thus, we assume that $n(p = C)$ is identical to the set $\{\cdots, n(p = C, o = 4), n(p = C, o = 5), n(p = C, o = 6), \cdots\}$. This way of thinking is closely related to an extensional representation of a class being a set of its instances. Furthermore, $n(p = C, o = \{\cdots, 4, 5, 6, \cdots\})$ can be also regarded identical to $n(p = C)$. The ordering of sets is summarized as

$$\begin{aligned} & n(p = C, o = 5) \\ \sqsubseteq\ & n(p = C) \\ =\ & \{\cdots, n(p = C, o = 4), n(p = C, o = 5), \\ & \quad n(p = C, o = 6), \cdots\} \\ =\ & n(p = C, o = \{\cdots, 4, 5, 6, \cdots\}). \end{aligned}$$

As you probably noticed, with respect to sets, $\sqsubseteq$ works the same as $\subseteq$ (set inclusion). Moreover, we can naturally introduce the deductive rule

$$v \sqsubseteq w \rightarrow o(a = v) \sqsubseteq o(a = w).$$

In this example, from $5 \sqsubseteq \{\cdots, 4, 5, 6, \cdots\}$, $n(p = C, o = 5) \sqsubseteq n(p = C, o = \{\cdots, 4, 5, 6, \cdots\})$ is obtained.

3.2 Chord

A chord is, in general, characterized by its subchords. For instance, the sound of a C_7 chord is characterized by its tritone ($E - B^\flat$); a fractional chord is characterized by its numerator and denominator subchords. Since an object having only partial information is more abstract, subchords are more abstract than the entire chord that includes them. To represent this abstraction relation, we therefore introduce **a μ term** with elements $e_1, e_2, \cdots$ denoted by $[e_1, e_2, \cdots]$. Using μ terms, the above example of the C_7 chord abstraction is described as

$$\begin{aligned} & [n(p = C), n(p = E), n(p = G), n(p = B^\flat)] \\ & \quad \sqsubseteq [n(p = E), n(p = B^\flat)]. \end{aligned}$$

The $\sqsubseteq$-ordering of μ terms is inverse to that of sets. Sets and μ terms can be nested arbitrarily.

Next, we define the other properties of μ terms. Like a note object, an entire μ term including larger elements is larger; i.e., the following rule is assumed

$$v \sqsubseteq w \rightarrow [\cdots, v, \cdots] \sqsubseteq [\cdots, w, \cdots].$$

Element positions of a μ term is ordered unlike a set or the arguments of a note object. For instance, there is no

[1] A harmony is composed of two or more adjacent 3rd intervals.

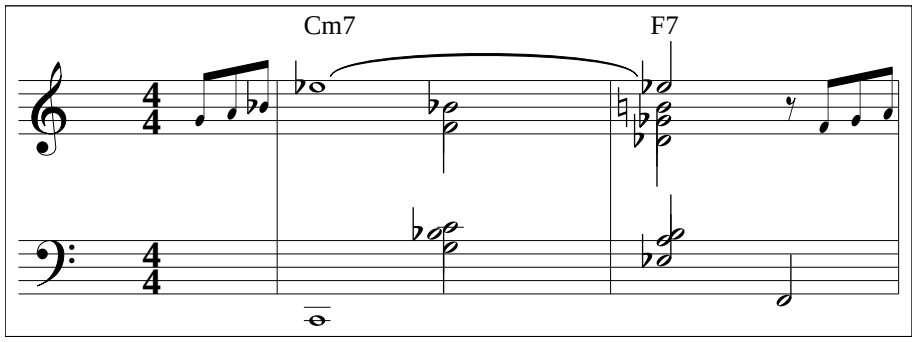

Figure 1: The First Two Bars of Autumn Leaves

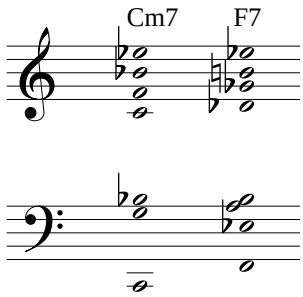

Figure 2: Supplementary Score

$\sqsubseteq$-ordering between $[a,b]$ and $[b,a]$ (if $a \neq b$). The abstraction by time grouping and removing is represented by μ terms as follows

$$\begin{array}{l}[[n(p=C,o=5,t=7)],\\ \ [n(p=E,o=5,t=13),n(p=G,o=5,t=11),\\ \ \ n(p=B^\flat,o=5,t=13)]]\\ \quad \sqsubseteq [[n(p=E,o=5,t=\{11,13\}),\\ \qquad n(p=G,o=5,t=\{11,13\}),\\ \qquad n(p=B^\flat,o=5,t=\{11,13\})]]\\ \quad \sqsubseteq [[n(p=E,o=5),n(p=G,o=5),\\ \qquad n(p=B^\flat,o=5)]]\end{array}$$

3.3 Demonstration

This section demonstrates the abstraction from Figure 1 to Figure 2, using note objects, μ terms and the $\sqsubseteq$-ordering.

First, the μ term (1) in Figure 3 stands for the actual solo performance shown in Figure 1. For space efficiency, $n(P,O,T,D)$ is an abbreviation of $n(p=P,o=O,t=T,d=D)$. For simplicity, the duration of an 8th note is supposed to be 1 clock unit and ties are not taken into account. Next, abstraction by removing the parts of melody leads to (2) in Figure 3. Next, abstraction by time grouping leads to (3) in Figure 3. Finally, abstraction by relativizing leads to (4) in Figure 3, which corresponds to the supplementary score in Figure 2. Here, $n(P,O)$ is an abbreviation of $n(p=P,o=O)$. It is important that (1) $\sqsubseteq$ (2) $\sqsubseteq$ (3) $\sqsubseteq$ (4) holds in Figure 3.

4 Association

The analysis of jazz piano performance consists of association as well as abstraction. The association is to relate different musical concepts represented by objects to each other. It is implemented by attributes. For instance, consider an example in which the chord name of a μ term M_0 is G_7. An object corresponding to a chord name is preliminarily defined as $cn(root=G,name=7)$. Moreover, a new notation useful for designating attribute values is introduced; it is called a dotted notation, denoted as $X.a=v$, which means $X \equiv o(a=v)$. Our method connects M_0 and $cn(root=G,name=7)$ using attributes in the following way.

$$\begin{array}{l}M_0.chord_name = cn(root=G,name=7)\\ cn(root=G,name=7,instance=\{M_0\})\end{array}$$

This means that the chord name of M_0 is G_7 and that an instance of G_7 is M_0.

4.1 Voice Leading

This method of forming associations by attributes is applied to the sample problem. Suppose

$$\begin{array}{l}M_1=[n(B^\flat,5),n(F,5),n(C,5),n(B^\flat,4)],\\ M_2=[n(B,5),n(G^\flat,5),n(D^\flat,5),n(B,4)].\end{array}$$

The comment quoted in Section 2 states voice leading as "Some of the inside voices of C_{m7} move to the inside voices of the subsequent F_7 chord by a semitone up". Our method translates this part as

$$(4) \sqsubseteq [M_1,M_2] \ \text{ and } \ [M_1,M_2].diff = 1.$$

That is, for every pair of notes occurring in M_1 and M_2, the pitch difference is 1 semitone.

4.2 Chord Structure

Suppose

$$\begin{array}{l}M_3=[n(E^\flat,6),n(B^\flat,5),n(F,5),n(C,5)],\\ M_4=[n(E^\flat,6),n(B^\flat,5),n(F,5),n(C,5),\\ \qquad n(B^\flat,4),n(G,4),n(C,3)].\end{array}$$

In terms of voicing, the comment also states that C_{m7} has three 4th intervals. Our method translates the comment to

$$\begin{array}{l}M_4.chord_name = cn(root=C,name=m7),\\ M_4 \sqsubseteq M_3 \ and \ M_3.interval = 5.\end{array}$$

That is, every pitch difference of adjacent notes in M_3 that is a subchord of C_{m7} is 5 semitones (4th degree). Moreover, the tension notes of C_{m7} and F_7 in (4) are $11th$ and $\flat 9, \#11, \flat 13ths$, respectively. Then, suppose $(4) \equiv [A,B]$, and the statements in terms of tension notes are translated to

$$\begin{array}{lcl}A.tension & = & \{11\}\\ B.tension & = & \{\flat 9, \#11, \flat 13\}.\end{array}$$

5 Concluding Remarks

As mentioned earlier, a transcription from an actual performance by a jazz pianist, its explanation, and analysis to the performance are given as the starting point of this work. If the jazz piano knowledge included in

$$
\begin{array}{ll}
[& [n(G,5,-3,1), n(A,5,-2,1), n(B^\flat,5,-1,1)], \\
& [n(E^\flat,6,0,8), n(C,3,0,8)], \\
& [n(B^\flat,5,4,4), n(F,5,4,4), n(C,5,4,4), n(B^\flat,4,4,4), n(G,4,4,4)], \\
& [n(E^\flat,6,8,4), n(B,5,8,4), n(G^\flat,5,8,4), n(D^\flat,5,8,4), n(B,4,8,4), n(A,4,8,4), n(E^\flat,4,8,4)], \\
& [n(F,3,12,4)], \\
& [n(F,5,13,1), n(G,5,14,1), n(A,5,15,1)] \ ]
\end{array} \tag{1}
$$

$$
\begin{array}{ll}
[& [n(E^\flat,6,0,8), n(C,3,0,8)], \\
& [n(B^\flat,5,4,4), n(F,5,4,4), n(C,5,4,4), n(B^\flat,4,4,4), n(G,4,4,4)], \\
& [n(E^\flat,6,8,4), n(B,5,8,4), n(G^\flat,5,8,4), n(D^\flat,5,8,4), n(B,4,8,4), n(A,4,8,4), n(E^\flat,4,8,4)], \\
& [n(F,3,12,4)] \ ]
\end{array} \tag{2}
$$

$$
\begin{array}{ll}
[& [n(E^\flat,6,\{0,4\},8), n(C,3,\{0,4\},8), \\
& \ n(B^\flat,5,\{0,4\},4), n(F,5,\{0,4\},4), n(C,5,\{0,4\},4), n(B^\flat,4,\{0,4\},4), n(G,4,\{0,4\},4)], \\
& [n(E^\flat,6,\{8,12\},4), n(B,5,\{8,12\},4), n(G^\flat,5,\{8,12\},4), n(D^\flat,5,\{8,12\},4), \\
& \ n(B,4,\{8,12\},4), n(A,4,\{8,12\},4), n(E^\flat,4,\{8,12\},4), n(F,3,\{8,12\},4)] \ ]
\end{array} \tag{3}
$$

$$
\begin{array}{ll}
[& [n(E^\flat,6), n(C,3), n(B^\flat,5), n(F,5), n(C,5), n(B^\flat,4), n(G,4)], \\
& [n(E^\flat,6), n(B,5), n(G^\flat,5), n(D^\flat,5), n(B,4), n(A,4), n(E^\flat,4), n(F,3)] \ ]
\end{array} \tag{4}
$$

Figure 3: Demonstration: Abstraction of Actual Performance

these materials is put into a knowledge base by using the method presented in this paper, a user can issue queries and execute various operations, such as (deductive) inference, searching, updating and inheritance along the $\sqsubseteq$-ordering.

Two example queries are shown below, although the format is tentative.

Query 1: The question "What is the chord name of a chord which includes a subchord having two 4th intervals" is translated into

$$
\begin{array}{l}
[N_1, N_2, N_3].\mathit{diff} = 5 \wedge \\
X \sqsubseteq [N_1, N_2, N_3] \wedge \\
X.\mathit{chord_name} = \mathit{Answer}.
\end{array}
$$

Query 2: The question "Show the real voicing of the IIm7-V7 motion" is translated into

$$
\begin{array}{l}
X.\mathit{chord_name} = A \wedge A.\mathit{name} = m7 \wedge \\
A.\mathit{root} - [X,Y].\mathit{key} = 2 \wedge \\
Y.\mathit{chord_name} = B \wedge B.\mathit{name} = 7 \wedge \\
B.\mathit{root} - [X,Y].\mathit{key} = 7 \wedge \\
\mathit{Answer} \sqsubseteq [X,Y].
\end{array}
$$

Almost all conventional musical knowledge-base systems store only the information abstracted from examples and text books in the form of rules. An advantage of this approach is that retrieval operations are efficient; in a sense, the abstraction can be regarded as the preprocessing of stored data. However, the information is inevitably lost during abstraction, since we cannot know what kinds of information will be required to answer unknown queries beforehand. Lack of necessary information for answering queries leads to ambiguous and possibly incorrect results. Thus, our musical knowledge-base system that is being developed at present stores every object, i.e. both abstracted scores and a source score, structured by the $\sqsubseteq$-ordering. This may cause heavier retrieval overhead, but there is no loss of information and proper answers can be generated.

The analysis by abstraction and association will be automated to some extent. The current system is supposed to interactively and incrementally manipulate jazz piano knowledge, especially harmony theory for jazz solo piano. In future, the application area will be expanded to other musical theories.

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