

## 課題と解答

### [課題 5-1] (30 点)

以下をスコールム化しなさい .

- $(\forall y)(\forall z)(\exists u)(\forall v)(\exists w)(\exists x)P(x, y, z, u, v, w)$
- $(\exists x)(\forall y)(\forall z)(\exists u)(\forall v)(\exists w)P(x, y, z, u, v, w)$
- $(\forall x)(\forall y)(\exists z)(\forall w)(\exists u)(P(x, y) \vee (Q(z, w) \wedge R(u)))$

(解答)

以下で  $f, g, h$  は関数記号であるが , 他の関数記号でも良い .

$$\begin{aligned} & (\forall y)(\forall z)(\exists u)(\forall v)(\exists w)(\exists x)P(x, y, z, u, v, w) \\ &= (\forall y)(\forall z)(\forall v)P(h(y, z, v), y, z, f(y, z), v, g(y, z, v)) \end{aligned}$$

$$\begin{aligned} & (\exists x)(\forall y)(\forall z)(\exists u)(\forall v)(\exists w)P(x, y, z, u, v, w) \\ &= (\forall y)(\forall z)(\forall v)P(a, y, z, f(y, z), v, g(y, z, v)) \end{aligned}$$

$$\begin{aligned} & (\forall x)(\forall y)(\exists z)(\forall w)(\exists u)(P(x, y) \vee (Q(z, w) \wedge R(u))) \\ &= (\forall x)(\forall y)(\exists z)(\forall w)(\exists u)((P(x, y) \vee Q(z, w)) \wedge (P(x, y) \vee R(u))) \\ &= (\forall x)(\forall y)(\forall w)((P(x, y) \vee Q(f(x, y), w)) \wedge (P(x, y) \vee R(g(x, y, w)))) \end{aligned}$$

### [課題 5-2] (20 点)

以下の論理式を節集合に変換しなさい .

$$(\forall x)(P(x) \rightarrow (\exists y)((Q(x, y) \rightarrow P(a)) \wedge (\forall z)(Q(y, z) \rightarrow P(x))))$$

(解答)

まずスコールム化を行う .

$$\begin{aligned} & (\forall x)(P(x) \rightarrow (\exists y)((Q(x, y) \rightarrow P(a)) \wedge (\forall z)(Q(y, z) \rightarrow P(x)))) \\ &= (\forall x)(P(x) \rightarrow (\exists y)((\neg Q(x, y) \vee P(a)) \wedge (\forall z)(\neg Q(y, z) \vee P(x)))) \\ &= (\forall x)(\neg P(x) \vee (\exists y)((\neg Q(x, y) \vee P(a)) \wedge (\forall z)(\neg Q(y, z) \vee P(x)))) \\ &= (\forall x)(\exists y)(\forall z)((\neg P(x) \vee (\neg Q(x, y) \vee P(a))) \wedge (\neg P(x) \vee (\neg Q(y, z) \vee P(x)))) \end{aligned}$$

$$\begin{aligned} &= (\forall x)(\exists y)(\forall z)(\neg P(x) \vee (\neg Q(x, y) \vee P(a))) \quad (\because \neg P(x) \vee \neg Q(y, z) \vee P(x) = \mathbf{T} \text{ であるから}) \\ &= (\forall x)(\neg P(x) \vee \neg Q(x, f(x)) \vee P(a)) \end{aligned}$$

よって求める節集合は  $\{\{\neg P(x), \neg Q(x, f(x)), P(a)\}\}$